

Pensieve header: The \$k=2\$ building blocks.

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In[*]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\SL2Invariant"];
<< SL2Invariant.m
$QZipFail = True;
Block[{$k = 3}, atoms = {
  am -> ami,j->k, bm -> bmi,j->k, dm -> dmi,j->k, R -> Ri,j, R̄ -> R̄i,j, P -> Pi,j,
  aS -> aSi, aS̄ -> aS̄i, bS -> bSi, bS̄ -> bS̄i, dS -> dSi, aΔ -> aΔi->j,k, bΔ -> bΔi->j,k,
  dΔ -> dΔi->j,k, C -> Ci, C̄ -> C̄i, Kink -> Kinki, Kink̄ -> Kink̄i, b2t -> b2ti, t2b -> t2bi
}] //
Column

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Loading KnotTheory` version of January 20, 2015, 10:42:19.1122.

Read more at <http://katlas.org/wiki/KnotTheory>.

This is Profile.m of <http://www.drorbn.net/AcademicPensieve/Projects/Profile/>.

This version: June 2018. Original version: July 1994.

$$am \rightarrow \mathbb{E}_{\{i,j\} \rightarrow \{k\}} \left[ a_k (\alpha_i + \alpha_j), x_k (e^{-\gamma \alpha_j} \xi_i + \xi_j), 1 \right]$$

$$bm \rightarrow \mathbb{E}_{\{i,j\} \rightarrow \{k\}} \left[ b_k (\beta_i + \beta_j), y_k (\eta_i + \eta_j), \right.$$

$$\left. 1 - y_k \beta_i \eta_j \epsilon + \frac{1}{2} (y_k \beta_i^2 \eta_j + y_k^2 \beta_i^2 \eta_j^2) \epsilon^2 + \frac{1}{6} (-y_k \beta_i^3 \eta_j - 3 y_k^2 \beta_i^3 \eta_j^2 - y_k^3 \beta_i^3 \eta_j^3) \epsilon^3 + \mathbf{0}[\epsilon]^4 \right]$$



$$R \rightarrow E_{\{\} \rightarrow \{i, j\}} \left[ \hbar a_j b_i, \hbar x_j y_i, 1 - \frac{1}{4} (\gamma \hbar^3 x_j^2 y_i^2) \in + \left( \frac{1}{9} \gamma^2 \hbar^5 x_j^3 y_i^3 + \frac{1}{32} \gamma^2 \hbar^6 x_j^4 y_i^4 \right) \in^2 + \right. \\ \left. \left( \frac{1}{48} \gamma^3 \hbar^5 x_j^2 y_i^2 - \frac{1}{16} \gamma^3 \hbar^7 x_j^4 y_i^4 - \frac{1}{36} \gamma^3 \hbar^8 x_j^5 y_i^5 - \frac{1}{384} \gamma^3 \hbar^9 x_j^6 y_i^6 \right) \in^3 + O[\epsilon]^4 \right]$$

$$\bar{R} \rightarrow E_{\{\} \rightarrow \{i, j\}} \left[ -\hbar a_j b_i, -\frac{\hbar x_j y_i}{B_i}, 1 + \left( -\frac{\hbar^2 a_j x_j y_i}{B_i} - \frac{3 \gamma \hbar^3 x_j^2 y_i^2}{4 B_i^2} \right) \in + \right. \\ \left( -\frac{\hbar^3 a_j^2 x_j y_i}{2 B_i} + \frac{\gamma^2 \hbar^4 x_j^2 y_i^2}{2 B_i^2} - \frac{3 \gamma \hbar^4 a_j x_j^2 y_i^2}{2 B_i^2} + \frac{\hbar^4 a_j^2 x_j^2 y_i^2}{2 B_i^2} - \frac{10 \gamma^2 \hbar^5 x_j^3 y_i^3}{9 B_i^3} + \frac{3 \gamma \hbar^5 a_j x_j^3 y_i^3}{4 B_i^3} + \frac{9 \gamma^2 \hbar^6 x_j^4 y_i^4}{32 B_i^4} \right) \in^2 + \\ \left( -\frac{\hbar^4 a_j^3 x_j y_i}{6 B_i} - \frac{3 \gamma^3 \hbar^5 x_j^2 y_i^2}{16 B_i^2} + \frac{\gamma^2 \hbar^5 a_j x_j^2 y_i^2}{B_i^2} - \frac{3 \gamma \hbar^5 a_j^2 x_j^2 y_i^2}{2 B_i^2} + \frac{\hbar^5 a_j^3 x_j^2 y_i^2}{2 B_i^2} + \frac{2 \gamma^3 \hbar^6 x_j^3 y_i^3}{B_i^3} - \frac{23 \gamma^2 \hbar^6 a_j x_j^3 y_i^3}{6 B_i^3} + \frac{15 \gamma \hbar^6 a_j^2 x_j^3 y_i^3}{8 B_i^3} - \right. \\ \left. \frac{\hbar^6 a_j^3 x_j^3 y_i^3}{6 B_i^3} - \frac{41 \gamma^3 \hbar^7 x_j^4 y_i^4}{16 B_i^4} + \frac{161 \gamma^2 \hbar^7 a_j x_j^4 y_i^4}{72 B_i^4} - \frac{3 \gamma \hbar^7 a_j^2 x_j^4 y_i^4}{8 B_i^4} + \frac{5 \gamma^3 \hbar^8 x_j^5 y_i^5}{6 B_i^5} - \frac{9 \gamma^2 \hbar^8 a_j x_j^5 y_i^5}{32 B_i^5} - \frac{9 \gamma^3 \hbar^9 x_j^6 y_i^6}{128 B_i^6} \right) \in^3 + O[\epsilon]^4 \left. \right]$$

$$P \rightarrow E_{\{i, j\} \rightarrow \{\}} \left[ \frac{\alpha_j \beta_i}{\hbar}, \frac{\eta_i \xi_j}{\hbar}, 1 + \frac{\gamma \eta_i^2 \xi_j^2 \epsilon}{4 \hbar} + \frac{(36 \gamma^2 \hbar^2 \eta_i^2 \xi_j^2 + 40 \gamma^2 \hbar \eta_i^3 \xi_j^3 + 9 \gamma^2 \eta_i^4 \xi_j^4) \epsilon^2}{288 \hbar^2} + \right. \\ \left. \left( \frac{1}{24} \gamma^3 \hbar \eta_i^2 \xi_j^2 + \frac{1}{6} \gamma^3 \eta_i^3 \xi_j^3 + \frac{13 \gamma^3 \eta_i^4 \xi_j^4}{96 \hbar} + \frac{5 \gamma^3 \eta_i^5 \xi_j^5}{144 \hbar^2} + \frac{\gamma^3 \eta_i^6 \xi_j^6}{384 \hbar^3} \right) \in^3 + O[\epsilon]^4 \right]$$

aS →

$$E_{\{i\} \rightarrow \{i\}} \left[ -a_i \alpha_i, -x_i \mathcal{A}_i \xi_i, 1 + \left( -\hbar a_i x_i \mathcal{A}_i \xi_i - \frac{1}{2} \gamma \hbar x_i^2 \mathcal{A}_i^2 \xi_i^2 \right) \in + \left( -\frac{1}{2} \hbar^2 a_i^2 x_i \mathcal{A}_i \xi_i + \frac{1}{4} \gamma^2 \hbar^2 x_i^2 \mathcal{A}_i^2 \xi_i^2 - \right. \right. \\ \left. \left. \gamma \hbar^2 a_i x_i^2 \mathcal{A}_i^2 \xi_i^2 + \frac{1}{2} \hbar^2 a_i^2 x_i^2 \mathcal{A}_i^2 \xi_i^2 - \frac{1}{2} \gamma^2 \hbar^2 x_i^3 \mathcal{A}_i^3 \xi_i^3 + \frac{1}{2} \gamma \hbar^2 a_i x_i^3 \mathcal{A}_i^3 \xi_i^3 + \frac{1}{8} \gamma^2 \hbar^2 x_i^4 \mathcal{A}_i^4 \xi_i^4 \right) \in^2 + \right. \\ \left( -\frac{1}{6} \hbar^3 a_i^3 x_i \mathcal{A}_i \xi_i - \frac{1}{12} \gamma^3 \hbar^3 x_i^2 \mathcal{A}_i^2 \xi_i^2 + \frac{1}{2} \gamma^2 \hbar^3 a_i x_i^2 \mathcal{A}_i^2 \xi_i^2 - \gamma \hbar^3 a_i^2 x_i^2 \mathcal{A}_i^2 \xi_i^2 + \right. \\ \left. \frac{1}{2} \hbar^3 a_i^3 x_i^2 \mathcal{A}_i^2 \xi_i^2 + \frac{2}{3} \gamma^3 \hbar^3 x_i^3 \mathcal{A}_i^3 \xi_i^3 - \frac{7}{4} \gamma^2 \hbar^3 a_i x_i^3 \mathcal{A}_i^3 \xi_i^3 + \frac{5}{4} \gamma \hbar^3 a_i^2 x_i^3 \mathcal{A}_i^3 \xi_i^3 - \right. \\ \left. \frac{1}{6} \hbar^3 a_i^3 x_i^3 \mathcal{A}_i^3 \xi_i^3 - \frac{19}{24} \gamma^3 \hbar^3 x_i^4 \mathcal{A}_i^4 \xi_i^4 + \gamma^2 \hbar^3 a_i x_i^4 \mathcal{A}_i^4 \xi_i^4 - \frac{1}{4} \gamma \hbar^3 a_i^2 x_i^4 \mathcal{A}_i^4 \xi_i^4 + \right. \\ \left. \frac{1}{4} \gamma^3 \hbar^3 x_i^5 \mathcal{A}_i^5 \xi_i^5 - \frac{1}{8} \gamma^2 \hbar^3 a_i x_i^5 \mathcal{A}_i^5 \xi_i^5 - \frac{1}{48} \gamma^3 \hbar^3 x_i^6 \mathcal{A}_i^6 \xi_i^6 \right) \in^3 + O[\epsilon]^4 \left. \right]$$

$$\bar{aS} \rightarrow E_{\{i\} \rightarrow \{i\}} \left[ -a_i \alpha_i, -x_i \mathcal{A}_i \xi_i, 1 + \left( \gamma \hbar x_i \mathcal{A}_i \xi_i - \hbar a_i x_i \mathcal{A}_i \xi_i - \frac{1}{2} \gamma \hbar x_i^2 \mathcal{A}_i^2 \xi_i^2 \right) \in + \right. \\ \frac{1}{8} \left( -4 \gamma^2 \hbar^2 x_i \mathcal{A}_i \xi_i + 8 \gamma \hbar^2 a_i x_i \mathcal{A}_i \xi_i - 4 \hbar^2 a_i^2 x_i \mathcal{A}_i \xi_i + 14 \gamma^2 \hbar^2 x_i^2 \mathcal{A}_i^2 \xi_i^2 - \right. \\ \left. 16 \gamma \hbar^2 a_i x_i^2 \mathcal{A}_i^2 \xi_i^2 + 4 \hbar^2 a_i^2 x_i^2 \mathcal{A}_i^2 \xi_i^2 - 8 \gamma^2 \hbar^2 x_i^3 \mathcal{A}_i^3 \xi_i^3 + 4 \gamma \hbar^2 a_i x_i^3 \mathcal{A}_i^3 \xi_i^3 + \gamma^2 \hbar^2 x_i^4 \mathcal{A}_i^4 \xi_i^4 \right) \in^2 + \\ \left( \frac{1}{6} \gamma^3 \hbar^3 x_i \mathcal{A}_i \xi_i - \frac{1}{2} \gamma^2 \hbar^3 a_i x_i \mathcal{A}_i \xi_i + \frac{1}{2} \gamma \hbar^3 a_i^2 x_i \mathcal{A}_i \xi_i - \frac{1}{6} \hbar^3 a_i^3 x_i \mathcal{A}_i \xi_i - \frac{25}{12} \gamma^3 \hbar^3 x_i^2 \mathcal{A}_i^2 \xi_i^2 + \right. \\ \left. 4 \gamma^2 \hbar^3 a_i x_i^2 \mathcal{A}_i^2 \xi_i^2 - \frac{5}{2} \gamma \hbar^3 a_i^2 x_i^2 \mathcal{A}_i^2 \xi_i^2 + \frac{1}{2} \hbar^3 a_i^3 x_i^2 \mathcal{A}_i^2 \xi_i^2 + \frac{23}{6} \gamma^3 \hbar^3 x_i^3 \mathcal{A}_i^3 \xi_i^3 - \frac{19}{4} \gamma^2 \hbar^3 a_i x_i^3 \mathcal{A}_i^3 \xi_i^3 + \right. \\ \left. \frac{7}{4} \gamma \hbar^3 a_i^2 x_i^3 \mathcal{A}_i^3 \xi_i^3 - \frac{1}{6} \hbar^3 a_i^3 x_i^3 \mathcal{A}_i^3 \xi_i^3 - \frac{49}{24} \gamma^3 \hbar^3 x_i^4 \mathcal{A}_i^4 \xi_i^4 + \frac{3}{2} \gamma^2 \hbar^3 a_i x_i^4 \mathcal{A}_i^4 \xi_i^4 - \right. \\ \left. \frac{1}{4} \gamma \hbar^3 a_i^2 x_i^4 \mathcal{A}_i^4 \xi_i^4 + \frac{3}{8} \gamma^3 \hbar^3 x_i^5 \mathcal{A}_i^5 \xi_i^5 - \frac{1}{8} \gamma^2 \hbar^3 a_i x_i^5 \mathcal{A}_i^5 \xi_i^5 - \frac{1}{48} \gamma^3 \hbar^3 x_i^6 \mathcal{A}_i^6 \xi_i^6 \right) \in^3 + O[\epsilon]^4 \left. \right]$$

$$bS \rightarrow E_{\{i\} \rightarrow \{i\}} \left[ -b_i \beta_i, -\frac{y_i \eta_i}{B_i}, \right.$$

$$1 + \left( -\frac{y_i \beta_i \eta_i}{B_i} - \frac{\gamma \hbar y_i^2 \eta_i^2}{2 B_i^2} \right) \in + \left( -\frac{y_i \beta_i^2 \eta_i}{2 B_i} + \frac{\gamma^2 \hbar^2 y_i^2 \eta_i^2}{4 B_i^2} - \frac{\gamma \hbar y_i^2 \beta_i \eta_i}{B_i^2} + \frac{y_i^2 \beta_i^2 \eta_i^2}{2 B_i^2} - \frac{\gamma^2 \hbar^2 y_i^3 \eta_i^3}{2 B_i^3} + \frac{\gamma \hbar y_i^3 \beta_i \eta_i}{2 B_i^3} + \frac{\gamma^2 \hbar^2 y_i^4 \eta_i^4}{8 B_i^4} \right) \in^2 + \\ \left( -\frac{y_i \beta_i^3 \eta_i}{6 B_i} - \frac{\gamma^3 \hbar^3 y_i^2 \eta_i^2}{12 B_i^2} + \frac{\gamma^2 \hbar^2 y_i^2 \beta_i \eta_i}{2 B_i^2} - \frac{\gamma \hbar y_i^2 \beta_i^2 \eta_i}{B_i^2} + \frac{y_i^2 \beta_i^3 \eta_i^2}{2 B_i^2} + \frac{2 \gamma^3 \hbar^3 y_i^3 \eta_i^3}{3 B_i^3} - \frac{7 \gamma^2 \hbar^2 y_i^3 \beta_i \eta_i}{4 B_i^3} + \frac{5 \gamma \hbar y_i^3 \beta_i^2 \eta_i}{4 B_i^3} - \right. \\ \left. \frac{y_i^3 \beta_i^3 \eta_i^2}{6 B_i^3} - \frac{19 \gamma^3 \hbar^3 y_i^4 \eta_i^4}{24 B_i^4} + \frac{\gamma^2 \hbar^2 y_i^4 \beta_i \eta_i}{B_i^4} - \frac{\gamma \hbar y_i^4 \beta_i^2 \eta_i}{4 B_i^4} + \frac{\gamma^3 \hbar^3 y_i^5 \eta_i^5}{4 B_i^5} - \frac{\gamma^2 \hbar^2 y_i^5 \beta_i \eta_i}{8 B_i^5} - \frac{\gamma^3 \hbar^3 y_i^6 \eta_i^6}{48 B_i^6} \right) \in^3 + O[\epsilon]^4 \left. \right]$$

$$\bar{bS} \rightarrow E_{\{i\} \rightarrow \{i\}} \left[ -b_i \beta_i, -\frac{y_i \eta_i}{B_i}, 1 + \left( \frac{\gamma \hbar y_i \eta_i}{B_i} - \frac{y_i \beta_i \eta_i}{B_i} - \frac{\gamma \hbar y_i^2 \eta_i^2}{2 B_i^2} \right) \in + \right.$$

$$\left( -\frac{\gamma^2 \hbar^2 y_i \eta_i}{2 B_i} + \frac{\gamma \hbar y_i \beta_i \eta_i}{B_i} - \frac{y_i \beta_i^2 \eta_i}{2 B_i} + \frac{7 \gamma^2 \hbar^2 y_i^2 \eta_i^2}{4 B_i^2} - \frac{2 \gamma \hbar y_i^2 \beta_i \eta_i}{B_i^2} + \frac{y_i^2 \beta_i^2 \eta_i^2}{2 B_i^2} - \frac{\gamma^2 \hbar^2 y_i^3 \eta_i^3}{B_i^3} + \frac{\gamma \hbar y_i^3 \beta_i \eta_i}{2 B_i^3} + \frac{\gamma^2 \hbar^2 y_i^4 \eta_i^4}{8 B_i^4} \right) \in^2 + \\ \left( \frac{\gamma^3 \hbar^3 y_i \eta_i}{6 B_i} - \frac{\gamma^2 \hbar^2 y_i \beta_i \eta_i}{2 B_i} + \frac{\gamma \hbar y_i \beta_i^2 \eta_i}{2 B_i} - \frac{y_i \beta_i^3 \eta_i}{6 B_i} - \frac{25 \gamma^3 \hbar^3 y_i^2 \eta_i^2}{12 B_i^2} + \frac{4 \gamma^2 \hbar^2 y_i^2 \beta_i \eta_i}{B_i^2} - \right. \\ \left. \frac{5 \gamma \hbar y_i^2 \beta_i^2 \eta_i}{2 B_i^2} + \frac{y_i^2 \beta_i^3 \eta_i^2}{2 B_i^2} + \frac{23 \gamma^3 \hbar^3 y_i^3 \eta_i^3}{6 B_i^3} - \frac{19 \gamma^2 \hbar^2 y_i^3 \beta_i \eta_i}{4 B_i^3} + \frac{7 \gamma \hbar y_i^3 \beta_i^2 \eta_i}{4 B_i^3} - \frac{y_i^3 \beta_i^3 \eta_i^2}{6 B_i^3} - \frac{49 \gamma^3 \hbar^3 y_i^4 \eta_i^4}{24 B_i^4} + \right. \\ \left. \frac{3 \gamma^2 \hbar^2 y_i^4 \beta_i \eta_i}{2 B_i^4} - \frac{\gamma \hbar y_i^4 \beta_i^2 \eta_i}{4 B_i^4} + \frac{3 \gamma^3 \hbar^3 y_i^5 \eta_i^5}{8 B_i^5} - \frac{\gamma^2 \hbar^2 y_i^5 \beta_i \eta_i}{8 B_i^5} - \frac{\gamma^3 \hbar^3 y_i^6 \eta_i^6}{48 B_i^6} \right) \in^3 + O[\epsilon]^4 \left. \right]$$

$$\begin{aligned}
 dS \rightarrow E_{\{i\} \rightarrow \{i\}} & \left[ -\mathbf{a}_i \alpha_i - \mathbf{b}_i \beta_i, -\frac{y_i \mathcal{A}_i \eta_i}{B_i} - \mathbf{x}_i \mathcal{A}_i \xi_i + \frac{(\mathcal{A}_i - B_i \mathcal{A}_i) \eta_i \xi_i}{h B_i}, \right. \\
 1 + & \left( \frac{\gamma h y_i \mathcal{A}_i \eta_i}{B_i} - \frac{y_i \mathcal{A}_i \beta_i \eta_i}{B_i} - \frac{\gamma h y_i^2 \mathcal{A}_i^2 \eta_i^2}{2 B_i^2} - h \mathbf{a}_i \mathbf{x}_i \mathcal{A}_i \xi_i - \mathbf{x}_i \mathcal{A}_i \beta_i \xi_i + \frac{\mathbf{a}_i \mathcal{A}_i \eta_i \xi_i}{B_i} - \right. \\
 & \frac{\gamma h \mathbf{x}_i y_i \mathcal{A}_i^2 \eta_i \xi_i}{B_i} + \frac{(-\gamma \mathcal{A}_i + \gamma B_i \mathcal{A}_i) \eta_i \xi_i}{B_i} + \frac{(\mathcal{A}_i - B_i \mathcal{A}_i) \beta_i \eta_i \xi_i}{h B_i} + \frac{y_i (3 \gamma \mathcal{A}_i^2 - \gamma B_i \mathcal{A}_i^2) \eta_i^2 \xi_i}{2 B_i^2} - \\
 & \left. \frac{1}{2} \gamma h \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^2 + \frac{\mathbf{x}_i (3 \gamma \mathcal{A}_i^2 - \gamma B_i \mathcal{A}_i^2) \eta_i \xi_i^2}{2 B_i} + \frac{(-3 \gamma \mathcal{A}_i^2 + 4 \gamma B_i \mathcal{A}_i^2 - \gamma B_i^2 \mathcal{A}_i^2) \eta_i^2 \xi_i^2}{4 h B_i^2} \right) \in + \\
 & \left( -\frac{\gamma^2 h^2 y_i \mathcal{A}_i \eta_i}{2 B_i} + \frac{\gamma h y_i \mathcal{A}_i \beta_i \eta_i}{B_i} - \frac{y_i \mathcal{A}_i \beta_i^2 \eta_i}{2 B_i} + \frac{7 \gamma^2 h^2 y_i^2 \mathcal{A}_i^2 \eta_i^2}{4 B_i^2} - \frac{2 \gamma h y_i^2 \mathcal{A}_i^2 \beta_i \eta_i^2}{B_i^2} + \frac{y_i^2 \mathcal{A}_i^2 \beta_i^2 \eta_i^2}{2 B_i^2} - \frac{\gamma^2 h^2 y_i^3 \mathcal{A}_i^3 \eta_i^3}{B_i^3} + \right. \\
 & \frac{\gamma h y_i^3 \mathcal{A}_i^3 \beta_i \eta_i^3}{2 B_i^3} + \frac{\gamma^2 h^2 y_i^4 \mathcal{A}_i^4 \eta_i^4}{8 B_i^4} - \frac{1}{2} h^2 \mathbf{a}_i^2 \mathbf{x}_i \mathcal{A}_i \xi_i - h \mathbf{a}_i \mathbf{x}_i \mathcal{A}_i \beta_i \xi_i - \frac{1}{2} \mathbf{x}_i \mathcal{A}_i \beta_i^2 \xi_i - \frac{\gamma h \mathbf{a}_i \mathcal{A}_i \eta_i \xi_i}{B_i} + \\
 & \frac{h \mathbf{a}_i^2 \mathcal{A}_i \eta_i \xi_i}{2 B_i} + \frac{3 \gamma^2 h^2 \mathbf{x}_i y_i \mathcal{A}_i^2 \eta_i \xi_i}{2 B_i} - \frac{2 \gamma h^2 \mathbf{a}_i \mathbf{x}_i y_i \mathcal{A}_i^2 \eta_i \xi_i}{B_i} + \frac{(\gamma^2 h \mathcal{A}_i - \gamma^2 h B_i \mathcal{A}_i) \eta_i \xi_i}{2 B_i} + \frac{\mathbf{a}_i \mathcal{A}_i \beta_i \eta_i \xi_i}{B_i} - \\
 & \frac{3 \gamma h \mathbf{x}_i y_i \mathcal{A}_i^2 \beta_i \eta_i \xi_i}{B_i} + \frac{h \mathbf{a}_i \mathbf{x}_i y_i \mathcal{A}_i^2 \beta_i \eta_i \xi_i}{B_i} + \frac{(-\gamma \mathcal{A}_i + \gamma B_i \mathcal{A}_i) \beta_i \eta_i \xi_i}{B_i} + \frac{\mathbf{x}_i y_i \mathcal{A}_i^2 \beta_i^2 \eta_i \xi_i}{B_i} + \frac{(\mathcal{A}_i - B_i \mathcal{A}_i) \beta_i^2 \eta_i \xi_i}{2 h B_i} + \\
 & \frac{5 \gamma h \mathbf{a}_i y_i \mathcal{A}_i^2 \eta_i^2 \xi_i}{2 B_i^2} - \frac{5 \gamma^2 h^2 \mathbf{x}_i y_i \mathcal{A}_i^2 \eta_i^2 \xi_i}{2 B_i^2} + \frac{\gamma h^2 \mathbf{a}_i \mathbf{x}_i y_i^2 \mathcal{A}_i^2 \eta_i^2 \xi_i}{2 B_i^2} + \frac{y_i (-21 \gamma^2 h \mathcal{A}_i^2 + 9 \gamma^2 h B_i \mathcal{A}_i^2) \eta_i^2 \xi_i}{4 B_i^2} - \frac{\mathbf{a}_i y_i \mathcal{A}_i^2 \beta_i \eta_i^2 \xi_i}{B_i^2} + \\
 & \frac{3 \gamma h \mathbf{x}_i y_i^2 \mathcal{A}_i^2 \beta_i \eta_i^2 \xi_i}{2 B_i^2} + \frac{y_i (5 \gamma \mathcal{A}_i^2 - 3 \gamma B_i \mathcal{A}_i^2) \beta_i \eta_i^2 \xi_i}{B_i^2} + \frac{y_i (-\mathcal{A}_i^2 + B_i \mathcal{A}_i^2) \beta_i^2 \eta_i^2 \xi_i}{h B_i^2} - \frac{\gamma h \mathbf{a}_i y_i^2 \mathcal{A}_i^2 \eta_i^2 \xi_i}{2 B_i^2} + \frac{\gamma^2 h^2 \mathbf{x}_i y_i^3 \mathcal{A}_i^3 \eta_i^3 \xi_i}{2 B_i^3} + \\
 & \frac{y_i^2 (14 \gamma^2 h \mathcal{A}_i^2 - 5 \gamma^2 h B_i \mathcal{A}_i^2) \eta_i^2 \xi_i}{3 B_i^3} + \frac{y_i^2 (-2 \gamma \mathcal{A}_i^2 + \gamma B_i \mathcal{A}_i^2) \beta_i \eta_i^2 \xi_i}{B_i^3} + \frac{y_i^2 (-3 \gamma^2 h \mathcal{A}_i^2 + \gamma^2 h B_i \mathcal{A}_i^2) \eta_i^3 \xi_i}{4 B_i^4} + \frac{1}{4} \gamma^2 h^2 \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^2 - \\
 & \gamma h^2 \mathbf{a}_i \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^2 + \frac{1}{2} h^2 \mathbf{a}_i^2 \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^2 - \gamma h \mathbf{x}_i^2 \mathcal{A}_i^2 \beta_i \xi_i^2 + h \mathbf{a}_i \mathbf{x}_i^2 \mathcal{A}_i^2 \beta_i \xi_i^2 + \frac{1}{2} \mathbf{x}_i^2 \mathcal{A}_i^2 \beta_i^2 \xi_i^2 - \frac{h \mathbf{a}_i^2 \mathbf{x}_i \mathcal{A}_i \eta_i \xi_i^2}{B_i} - \\
 & \frac{2 \gamma^2 h^2 \mathbf{x}_i^2 y_i \mathcal{A}_i^2 \eta_i \xi_i^2}{B_i} + \frac{\gamma h^2 \mathbf{a}_i \mathbf{x}_i^2 y_i \mathcal{A}_i^2 \eta_i \xi_i^2}{B_i} + \frac{\mathbf{a}_i \mathbf{x}_i (8 \gamma h \mathcal{A}_i^2 - 3 \gamma h B_i \mathcal{A}_i^2) \eta_i \xi_i^2}{2 B_i} + \frac{\mathbf{x}_i (-11 \gamma^2 h \mathcal{A}_i^2 + 3 \gamma^2 h B_i \mathcal{A}_i^2) \eta_i \xi_i^2}{4 B_i} + \\
 & \frac{3 \gamma h \mathbf{x}_i^2 y_i \mathcal{A}_i^2 \beta_i \eta_i \xi_i^2}{2 B_i} + \frac{\mathbf{a}_i \mathbf{x}_i (-2 \mathcal{A}_i^2 + B_i \mathcal{A}_i^2) \beta_i \eta_i \xi_i^2}{B_i} + \frac{\mathbf{x}_i (4 \gamma \mathcal{A}_i^2 - 2 \gamma B_i \mathcal{A}_i^2) \beta_i \eta_i \xi_i^2}{B_i} + \frac{\mathbf{x}_i (-\mathcal{A}_i^2 + B_i \mathcal{A}_i^2) \beta_i^2 \eta_i \xi_i^2}{h B_i} + \\
 & \frac{\mathbf{a}_i^2 \mathcal{A}_i^2 \eta_i^2 \xi_i^2}{2 B_i^2} + \frac{3 \gamma^2 h^2 \mathbf{x}_i^2 y_i^2 \mathcal{A}_i^2 \eta_i^2 \xi_i^2}{4 B_i^2} + \frac{\mathbf{a}_i (-5 \gamma \mathcal{A}_i^2 + 4 \gamma B_i \mathcal{A}_i^2) \eta_i^2 \xi_i^2}{2 B_i^2} + \frac{(21 \gamma^2 \mathcal{A}_i^2 - 30 \gamma^2 B_i \mathcal{A}_i^2 + 9 \gamma^2 B_i^2 \mathcal{A}_i^2) \eta_i^2 \xi_i^2}{8 B_i^2} + \\
 & \frac{\mathbf{a}_i \mathbf{x}_i y_i (-5 \gamma h \mathcal{A}_i^2 + \gamma h B_i \mathcal{A}_i^2) \eta_i^2 \xi_i^2}{2 B_i^2} + \frac{\mathbf{x}_i y_i (31 \gamma^2 h \mathcal{A}_i^2 - 11 \gamma^2 h B_i \mathcal{A}_i^2) \eta_i^2 \xi_i^2}{4 B_i^2} + \frac{\mathbf{a}_i (\mathcal{A}_i^2 - B_i \mathcal{A}_i^2) \beta_i \eta_i^2 \xi_i^2}{h B_i^2} + \\
 & \frac{(-5 \gamma \mathcal{A}_i^2 + 8 \gamma B_i \mathcal{A}_i^2 - 3 \gamma B_i^2 \mathcal{A}_i^2) \beta_i \eta_i^2 \xi_i^2}{2 h B_i^2} + \frac{\mathbf{x}_i y_i (-4 \gamma \mathcal{A}_i^2 + 2 \gamma B_i \mathcal{A}_i^2) \beta_i \eta_i^2 \xi_i^2}{B_i^2} + \frac{(\mathcal{A}_i^2 - 2 B_i \mathcal{A}_i^2 + B_i^2 \mathcal{A}_i^2) \beta_i^2 \eta_i^2 \xi_i^2}{2 h^2 B_i^2} + \\
 & \frac{\mathbf{a}_i y_i (3 \gamma \mathcal{A}_i^2 - \gamma B_i \mathcal{A}_i^2) \eta_i^2 \xi_i^2}{2 B_i^2} + \frac{y_i (-34 \gamma^2 \mathcal{A}_i^2 + 35 \gamma^2 B_i \mathcal{A}_i^2 - 7 \gamma^2 B_i^2 \mathcal{A}_i^2) \eta_i^2 \xi_i^2}{6 B_i^2} + \frac{\mathbf{x}_i y_i^2 (-9 \gamma^2 h \mathcal{A}_i^2 + 3 \gamma^2 h B_i \mathcal{A}_i^2) \eta_i^2 \xi_i^2}{4 B_i^2} + \\
 & \frac{y_i (9 \gamma \mathcal{A}_i^2 - 12 \gamma B_i \mathcal{A}_i^2 + 3 \gamma B_i^2 \mathcal{A}_i^2) \beta_i \eta_i^2 \xi_i^2}{4 h B_i^2} + \frac{y_i^2 (6 \gamma^2 \mathcal{A}_i^2 - 5 \gamma^2 B_i \mathcal{A}_i^2 + \gamma^2 B_i^2 \mathcal{A}_i^2) \eta_i^2 \xi_i^2}{4 B_i^2} - \frac{1}{2} \gamma^2 h^2 \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^3 + \\
 & \frac{1}{2} \gamma h^2 \mathbf{a}_i \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^3 + \frac{1}{2} \gamma h \mathbf{x}_i^2 \mathcal{A}_i^2 \beta_i \xi_i^3 + \frac{\gamma^2 h^2 \mathbf{x}_i^2 y_i \mathcal{A}_i^2 \eta_i \xi_i^3}{2 B_i} + \frac{\mathbf{a}_i \mathbf{x}_i^2 (-4 \gamma h \mathcal{A}_i^2 + \gamma h B_i \mathcal{A}_i^2) \eta_i \xi_i^3}{2 B_i} + \\
 & \frac{\mathbf{x}_i^2 (19 \gamma^2 h \mathcal{A}_i^2 - 7 \gamma^2 h B_i \mathcal{A}_i^2) \eta_i \xi_i^3}{6 B_i} + \frac{\mathbf{x}_i^2 (-2 \gamma \mathcal{A}_i^2 + \gamma B_i \mathcal{A}_i^2) \beta_i \eta_i \xi_i^3}{B_i} + \frac{\mathbf{a}_i \mathbf{x}_i (9 \gamma \mathcal{A}_i^2 - 6 \gamma B_i \mathcal{A}_i^2 + \gamma B_i^2 \mathcal{A}_i^2) \eta_i^2 \xi_i^3}{4 B_i^2} + \\
 & \frac{\mathbf{x}_i (-59 \gamma^2 \mathcal{A}_i^2 + 58 \gamma^2 B_i \mathcal{A}_i^2 - 11 \gamma^2 B_i^2 \mathcal{A}_i^2) \eta_i^2 \xi_i^3}{12 B_i^2} + \frac{\mathbf{x}_i^2 y_i (-9 \gamma^2 h \mathcal{A}_i^2 + 3 \gamma^2 h B_i \mathcal{A}_i^2) \eta_i^2 \xi_i^3}{4 B_i^2} + \frac{\mathbf{x}_i (9 \gamma \mathcal{A}_i^2 - 12 \gamma B_i \mathcal{A}_i^2 + 3 \gamma B_i^2 \mathcal{A}_i^2) \beta_i \eta_i^2 \xi_i^3}{4 h B_i^2} + \\
 & \frac{\mathbf{a}_i (-3 \gamma \mathcal{A}_i^2 + 4 \gamma B_i \mathcal{A}_i^2 - \gamma B_i^2 \mathcal{A}_i^2) \eta_i^2 \xi_i^3}{4 h B_i^2} + \frac{(34 \gamma^2 \mathcal{A}_i^2 - 69 \gamma^2 B_i \mathcal{A}_i^2 + 42 \gamma^2 B_i^2 \mathcal{A}_i^2 - 7 \gamma^2 B_i^3 \mathcal{A}_i^2) \eta_i^2 \xi_i^3}{18 h B_i^2} + \frac{\mathbf{x}_i y_i (6 \gamma^2 \mathcal{A}_i^2 - 5 \gamma^2 B_i \mathcal{A}_i^2 + \gamma^2 B_i^2 \mathcal{A}_i^2) \eta_i^2 \xi_i^3}{2 B_i^3} + \\
 & \frac{(-3 \gamma \mathcal{A}_i^2 + 7 \gamma B_i \mathcal{A}_i^2 - 5 \gamma B_i^2 \mathcal{A}_i^2 + \gamma B_i^3 \mathcal{A}_i^2) \beta_i \eta_i^2 \xi_i^3}{4 h^2 B_i^2} + \frac{y_i (-9 \gamma^2 \mathcal{A}_i^2 + 15 \gamma^2 B_i \mathcal{A}_i^2 - 7 \gamma^2 B_i^2 \mathcal{A}_i^2 + \gamma^2 B_i^3 \mathcal{A}_i^2) \eta_i^2 \xi_i^3}{8 h B_i^2} + \\
 & \frac{1}{8} \gamma^2 h^2 \mathbf{x}_i^4 \mathcal{A}_i^4 \xi_i^4 + \frac{\mathbf{x}_i^2 (-3 \gamma^2 h \mathcal{A}_i^2 + \gamma^2 h B_i \mathcal{A}_i^2) \eta_i \xi_i^4}{4 B_i} + \frac{\mathbf{x}_i^2 (6 \gamma^2 \mathcal{A}_i^2 - 5 \gamma^2 B_i \mathcal{A}_i^2 + \gamma^2 B_i^2 \mathcal{A}_i^2) \eta_i^2 \xi_i^4}{4 B_i^2} + \\
 & \left. \frac{\mathbf{x}_i (-9 \gamma^2 \mathcal{A}_i^2 + 15 \gamma^2 B_i \mathcal{A}_i^2 - 7 \gamma^2 B_i^2 \mathcal{A}_i^2 + \gamma^2 B_i^3 \mathcal{A}_i^2) \eta_i^2 \xi_i^4}{8 h B_i^2} + \frac{(9 \gamma^2 \mathcal{A}_i^2 - 24 \gamma^2 B_i \mathcal{A}_i^2 + 22 \gamma^2 B_i^2 \mathcal{A}_i^2 - 8 \gamma^2 B_i^3 \mathcal{A}_i^2 + \gamma^2 B_i^4 \mathcal{A}_i^2) \eta_i^2 \xi_i^4}{32 h^2 B_i^2} \right) \in 2 + \\
 & \left( \frac{\gamma^3 h^3 y_i \mathcal{A}_i \eta_i}{6 B_i} - \frac{\gamma^2 h^2 y_i \mathcal{A}_i \beta_i \eta_i}{2 B_i} + \frac{\gamma h y_i \mathcal{A}_i \beta_i^2 \eta_i}{2 B_i} - \frac{y_i \mathcal{A}_i \beta_i^3 \eta_i}{6 B_i} - \frac{25 \gamma^3 h^3 y_i^2 \mathcal{A}_i^2 \eta_i^2}{12 B_i^2} + \frac{4 \gamma^2 h^2 y_i^2 \mathcal{A}_i^2 \beta_i \eta_i^2}{B_i^2} - \frac{5 \gamma h y_i^2 \mathcal{A}_i^2 \beta_i^2 \eta_i^2}{2 B_i^2} - \right. \\
 & \frac{y_i^2 \mathcal{A}_i^2 \beta_i^3 \eta_i^2}{2 B_i^2} + \frac{23 \gamma^3 h^3 y_i^3 \mathcal{A}_i^3 \eta_i^3}{6 B_i^3} - \frac{19 \gamma^2 h^2 y_i^2 \mathcal{A}_i^2 \beta_i \eta_i^3}{4 B_i^2} + \frac{7 \gamma h y_i \mathcal{A}_i^2 \beta_i^2 \eta_i^3}{4 B_i^2} - \frac{y_i^2 \mathcal{A}_i^2 \beta_i^3 \eta_i^3}{6 B_i^2} - \frac{49 \gamma^3 h^3 y_i^4 \mathcal{A}_i^4 \eta_i^4}{24 B_i^4} + \frac{3 \gamma^2 h^2 y_i^4 \mathcal{A}_i^4 \beta_i \eta_i^4}{2 B_i^2} - \\
 & \frac{\gamma h y_i^4 \mathcal{A}_i^4 \beta_i^2 \eta_i^4}{4 B_i^4} + \frac{3 \gamma^3 h^3 y_i^5 \mathcal{A}_i^5 \eta_i^5}{8 B_i^5} - \frac{\gamma^2 h^2 y_i^5 \mathcal{A}_i^5 \beta_i \eta_i^5}{8 B_i^5} - \frac{\gamma^3 h^3 y_i^6 \mathcal{A}_i^6 \eta_i^6}{48 B_i^6} - \frac{1}{6} h^3 \mathbf{a}_i^3 \mathbf{x}_i \mathcal{A}_i \xi_i - \frac{1}{2} h^2 \mathbf{a}_i^2 \mathbf{x}_i \mathcal{A}_i \beta_i \xi_i - \\
 & \frac{1}{2} h \mathbf{a}_i \mathbf{x}_i \mathcal{A}_i \beta_i^2 \xi_i - \frac{1}{6} \mathbf{x}_i \mathcal{A}_i \beta_i^3 \xi_i + \frac{\gamma^2 h^2 \mathbf{a}_i \mathcal{A}_i \eta_i \xi_i}{2 B_i} - \frac{\gamma h^2 \mathbf{a}_i^2 \mathcal{A}_i \eta_i \xi_i}{2 B_i} + \frac{h^2 \mathbf{a}_i^3 \mathcal{A}_i \eta_i \xi_i}{6 B_i} - \frac{7 \gamma^3 h^3 \mathbf{x}_i y_i \mathcal{A}_i^2 \eta_i \xi_i}{6 B_i} + \\
 & \frac{2 \gamma^2 h^3 \mathbf{a}_i \mathbf{x}_i y_i \mathcal{A}_i^2 \eta_i \xi_i}{B_i} - \frac{\gamma h^3 \mathbf{a}_i^2 \mathbf{x}_i y_i \mathcal{A}_i^2 \eta_i \xi_i}{B_i} + \frac{(-\gamma^3 h^2 \mathcal{A}_i + \gamma^3 h^2 B_i \mathcal{A}_i) \eta_i \xi_i}{6 B_i} - \frac{\gamma h \mathbf{a}_i \mathcal{A}_i \beta_i \eta_i \xi_i}{2 B_i} + \frac{h \mathbf{a}_i^2 \mathcal{A}_i \beta_i \eta_i \xi_i}{2 B_i} + \\
 & \frac{7 \gamma^2 h^2 \mathbf{x}_i y_i \mathcal{A}_i^2 \beta_i \eta_i \xi_i}{2 B_i} - \frac{4 \gamma h^2 \mathbf{a}_i \mathbf{x}_i y_i \mathcal{A}_i^2 \beta_i \eta_i \xi_i}{B_i} + \frac{h^2 \mathbf{a}_i^2 \mathbf{x}_i y_i \mathcal{A}_i^2 \beta_i \eta_i \xi_i}{2 B_i} + \frac{(\gamma^2 h \mathcal{A}_i - \gamma^2 h B_i \mathcal{A}_i) \beta_i \eta_i \xi_i}{2 B_i} + \frac{\mathbf{a}_i \mathcal{A}_i \beta_i^2 \eta_i \xi_i}{2 B_i} - \\
 & \frac{7 \gamma h \mathbf{x}_i y_i \mathcal{A}_i^2 \beta_i \eta_i \xi_i}{2 B_i} + \frac{3 h \mathbf{a}_i \mathbf{x}_i y_i \mathcal{A}_i^2 \beta_i \eta_i \xi_i}{2 B_i} + \frac{(-\gamma \mathcal{A}_i + \gamma B_i \mathcal{A}_i) \beta_i^2 \eta_i \xi_i}{2 B_i} + \frac{\mathbf{x}_i y_i \mathcal{A}_i^2 \beta_i^2 \eta_i \xi_i}{B_i} + \frac{(\mathcal{A}_i - B_i \mathcal{A}_i) \beta_i^3 \eta_i \xi_i}{6 h B_i} - \\
 & \frac{23 \gamma^2 h^2 \mathbf{a}_i y_i \mathcal{A}_i^2 \eta_i^2 \xi_i}{4 B_i^2} + \frac{5 \gamma h^2 \mathbf{a}_i^2 y_i \mathcal{A}_i^2 \eta_i^2 \xi_i}{4 B_i^2} + \frac{7 \gamma^3 h^3 \mathbf{x}_i y_i^2 \mathcal{A}_i^2 \eta_i^2 \xi_i}{4 B_i^2} - \frac{17 \gamma^2 h^3 \mathbf{a}_i \mathbf{x}_i y_i^2 \mathcal{A}_i^2 \eta_i^2 \xi_i}{4 B_i^2} + \frac{\gamma h^3 \mathbf{a}_i^2 \mathbf{x}_i y_i^2 \mathcal{A}_i^2 \eta_i^2 \xi_i}{4 B_i^2} +
 \end{aligned}$$

$$\begin{aligned}
 a\Delta \rightarrow \mathbb{E}_{\{i\} \rightarrow \{j,k\}} & \left[ \mathbf{a}_j \alpha_i + \mathbf{a}_k \alpha_i, \mathbf{x}_j \xi_i + \mathbf{x}_k \xi_i, \right. \\
 & \mathbf{1} + \left( -\hbar \mathbf{a}_j \mathbf{x}_k \xi_i + \frac{1}{2} \gamma \hbar \mathbf{x}_j \mathbf{x}_k \xi_i^2 \right) \epsilon + \left( \frac{1}{2} \hbar^2 \mathbf{a}_j^2 \mathbf{x}_k \xi_i + \frac{1}{4} \gamma^2 \hbar^2 \mathbf{x}_j \mathbf{x}_k \xi_i^2 - \frac{1}{2} \gamma \hbar^2 \mathbf{a}_j \mathbf{x}_j \mathbf{x}_k \xi_i^2 + \right. \\
 & \quad \left. \frac{1}{2} \hbar^2 \mathbf{a}_j^2 \mathbf{x}_k^2 \xi_i^2 + \frac{1}{6} \gamma^2 \hbar^2 \mathbf{x}_j^2 \mathbf{x}_k \xi_i^3 + \frac{1}{6} \gamma^2 \hbar^2 \mathbf{x}_j \mathbf{x}_k^2 \xi_i^3 - \frac{1}{2} \gamma \hbar^2 \mathbf{a}_j \mathbf{x}_j \mathbf{x}_k^2 \xi_i^3 + \frac{1}{8} \gamma^2 \hbar^2 \mathbf{x}_j^2 \mathbf{x}_k^2 \xi_i^4 \right) \epsilon^2 + \\
 & \left( -\frac{1}{6} \hbar^3 \mathbf{a}_j^3 \mathbf{x}_k \xi_i + \frac{1}{12} \gamma^3 \hbar^3 \mathbf{x}_j \mathbf{x}_k \xi_i^2 - \frac{1}{4} \gamma^2 \hbar^3 \mathbf{a}_j \mathbf{x}_j \mathbf{x}_k \xi_i^2 + \frac{1}{4} \gamma \hbar^3 \mathbf{a}_j^2 \mathbf{x}_j \mathbf{x}_k \xi_i^2 - \frac{1}{2} \hbar^3 \mathbf{a}_j^2 \mathbf{x}_k^2 \xi_i^2 + \right. \\
 & \quad \frac{1}{6} \gamma^3 \hbar^3 \mathbf{x}_j^2 \mathbf{x}_k \xi_i^3 - \frac{1}{6} \gamma^2 \hbar^3 \mathbf{a}_j \mathbf{x}_j^2 \mathbf{x}_k \xi_i^3 + \frac{1}{6} \gamma^3 \hbar^3 \mathbf{x}_j \mathbf{x}_k^2 \xi_i^3 - \frac{7}{12} \gamma^2 \hbar^3 \mathbf{a}_j \mathbf{x}_j \mathbf{x}_k^2 \xi_i^3 + \\
 & \quad \frac{3}{4} \gamma \hbar^3 \mathbf{a}_j^2 \mathbf{x}_j \mathbf{x}_k^2 \xi_i^3 - \frac{1}{6} \hbar^3 \mathbf{a}_j^3 \mathbf{x}_k^3 \xi_i^3 + \frac{1}{24} \gamma^3 \hbar^3 \mathbf{x}_j^3 \mathbf{x}_k \xi_i^4 + \frac{1}{3} \gamma^3 \hbar^3 \mathbf{x}_j^2 \mathbf{x}_k^2 \xi_i^4 - \\
 & \quad \frac{5}{12} \gamma^2 \hbar^3 \mathbf{a}_j \mathbf{x}_j^2 \mathbf{x}_k^2 \xi_i^4 + \frac{1}{24} \gamma^3 \hbar^3 \mathbf{x}_j \mathbf{x}_k^3 \xi_i^4 - \frac{1}{6} \gamma^2 \hbar^3 \mathbf{a}_j \mathbf{x}_j \mathbf{x}_k^3 \xi_i^4 + \frac{1}{4} \gamma \hbar^3 \mathbf{a}_j^2 \mathbf{x}_j \mathbf{x}_k^3 \xi_i^4 + \\
 & \quad \left. \frac{1}{12} \gamma^3 \hbar^3 \mathbf{x}_j^3 \mathbf{x}_k^2 \xi_i^5 + \frac{1}{12} \gamma^3 \hbar^3 \mathbf{x}_j^2 \mathbf{x}_k^3 \xi_i^5 - \frac{1}{8} \gamma^2 \hbar^3 \mathbf{a}_j \mathbf{x}_j^2 \mathbf{x}_k^3 \xi_i^5 + \frac{1}{48} \gamma^3 \hbar^3 \mathbf{x}_j^3 \mathbf{x}_k^3 \xi_i^6 \right) \epsilon^3 + \mathbf{O}[\epsilon^4]
 \end{aligned}$$

$$\begin{aligned}
 b\Delta \rightarrow \mathbb{E}_{\{i\} \rightarrow \{j,k\}} & \left[ \mathbf{b}_j \beta_i + \mathbf{b}_k \beta_i, \mathbf{B}_k \mathbf{y}_j \eta_i + \mathbf{y}_k \eta_i, \mathbf{1} + \frac{1}{2} \gamma \hbar \mathbf{B}_k \mathbf{y}_j \mathbf{y}_k \eta_i^2 \epsilon + \right. \\
 & \left( \frac{1}{4} \gamma^2 \hbar^2 \mathbf{B}_k \mathbf{y}_j \mathbf{y}_k \eta_i^2 + \frac{1}{6} \gamma^2 \hbar^2 \mathbf{B}_k^2 \mathbf{y}_j^2 \mathbf{y}_k \eta_i^3 + \frac{1}{6} \gamma^2 \hbar^2 \mathbf{B}_k \mathbf{y}_j \mathbf{y}_k^2 \eta_i^3 + \frac{1}{8} \gamma^2 \hbar^2 \mathbf{B}_k^2 \mathbf{y}_j^2 \mathbf{y}_k^2 \eta_i^4 \right) \epsilon^2 + \\
 & \left( \frac{1}{12} \gamma^3 \hbar^3 \mathbf{B}_k \mathbf{y}_j \mathbf{y}_k \eta_i^2 + \frac{1}{6} \gamma^3 \hbar^3 \mathbf{B}_k^2 \mathbf{y}_j^2 \mathbf{y}_k \eta_i^3 + \frac{1}{6} \gamma^3 \hbar^3 \mathbf{B}_k \mathbf{y}_j \mathbf{y}_k^2 \eta_i^3 + \frac{1}{24} \gamma^3 \hbar^3 \mathbf{B}_k^3 \mathbf{y}_j^3 \mathbf{y}_k \eta_i^4 + \frac{1}{3} \gamma^3 \hbar^3 \mathbf{B}_k^2 \mathbf{y}_j^2 \mathbf{y}_k^2 \eta_i^4 + \right. \\
 & \quad \left. \frac{1}{24} \gamma^3 \hbar^3 \mathbf{B}_k \mathbf{y}_j \mathbf{y}_k^3 \eta_i^4 + \frac{1}{12} \gamma^3 \hbar^3 \mathbf{B}_k^2 \mathbf{y}_j^2 \mathbf{y}_k^2 \eta_i^5 + \frac{1}{12} \gamma^3 \hbar^3 \mathbf{B}_k \mathbf{y}_j^2 \mathbf{y}_k^3 \eta_i^5 + \frac{1}{48} \gamma^3 \hbar^3 \mathbf{B}_k^3 \mathbf{y}_j^3 \mathbf{y}_k^3 \eta_i^6 \right) \epsilon^3 + \mathbf{O}[\epsilon^4]
 \end{aligned}$$

$$\begin{aligned}
 d\Delta \rightarrow \mathbb{E}_{\{i\} \rightarrow \{j,k\}} & \left[ \mathbf{a}_j \alpha_i + \mathbf{a}_k \alpha_i + \mathbf{b}_j \beta_i + \mathbf{b}_k \beta_i, \right. \\
 & \mathbf{y}_j \eta_i + \mathbf{B}_j \mathbf{y}_k \eta_i + \mathbf{x}_j \xi_i + \mathbf{x}_k \xi_i, \mathbf{1} + \left( \frac{1}{2} \gamma \hbar \mathbf{B}_j \mathbf{y}_j \mathbf{y}_k \eta_i^2 - \hbar \mathbf{a}_j \mathbf{x}_k \xi_i + \frac{1}{2} \gamma \hbar \mathbf{x}_j \mathbf{x}_k \xi_i^2 \right) \epsilon + \\
 & \left( \frac{1}{4} \gamma^2 \hbar^2 \mathbf{B}_j \mathbf{y}_j \mathbf{y}_k \eta_i^2 + \frac{1}{6} \gamma^2 \hbar^2 \mathbf{B}_j \mathbf{y}_j^2 \mathbf{y}_k \eta_i^3 + \frac{1}{6} \gamma^2 \hbar^2 \mathbf{B}_j^2 \mathbf{y}_j \mathbf{y}_k^2 \eta_i^3 + \frac{1}{8} \gamma^2 \hbar^2 \mathbf{B}_j^2 \mathbf{y}_j^2 \mathbf{y}_k^2 \eta_i^4 + \frac{1}{2} \hbar^2 \mathbf{a}_j^2 \mathbf{x}_k \xi_i - \right. \\
 & \quad \frac{1}{2} \gamma \hbar^2 \mathbf{a}_j \mathbf{B}_j \mathbf{x}_k \mathbf{y}_j \mathbf{y}_k \eta_i^2 \xi_i + \frac{1}{4} \gamma^2 \hbar^2 \mathbf{x}_j \mathbf{x}_k \xi_i^2 - \frac{1}{2} \gamma \hbar^2 \mathbf{a}_j \mathbf{x}_j \mathbf{x}_k \xi_i^2 + \frac{1}{2} \hbar^2 \mathbf{a}_j^2 \mathbf{x}_k^2 \xi_i^2 + \\
 & \quad \left. \frac{1}{4} \gamma^2 \hbar^2 \mathbf{B}_j \mathbf{x}_j \mathbf{x}_k \mathbf{y}_j \mathbf{y}_k \eta_i^2 \xi_i^2 + \frac{1}{6} \gamma^2 \hbar^2 \mathbf{x}_j^2 \mathbf{x}_k \xi_i^3 + \frac{1}{6} \gamma^2 \hbar^2 \mathbf{x}_j \mathbf{x}_k^2 \xi_i^3 - \frac{1}{2} \gamma \hbar^2 \mathbf{a}_j \mathbf{x}_j \mathbf{x}_k^2 \xi_i^3 + \frac{1}{8} \gamma^2 \hbar^2 \mathbf{x}_j^2 \mathbf{x}_k^2 \xi_i^4 \right) \\
 & \epsilon^2 + \left( \frac{1}{12} \gamma^3 \hbar^3 \mathbf{B}_j \mathbf{y}_j \mathbf{y}_k \eta_i^2 + \frac{1}{6} \gamma^3 \hbar^3 \mathbf{B}_j \mathbf{y}_j^2 \mathbf{y}_k \eta_i^3 + \frac{1}{6} \gamma^3 \hbar^3 \mathbf{B}_j^2 \mathbf{y}_j \mathbf{y}_k^2 \eta_i^3 + \frac{1}{24} \gamma^3 \hbar^3 \mathbf{B}_j \mathbf{y}_j^3 \mathbf{y}_k \eta_i^4 + \right. \\
 & \quad \frac{1}{3} \gamma^3 \hbar^3 \mathbf{B}_j^2 \mathbf{y}_j^2 \mathbf{y}_k^2 \eta_i^4 + \frac{1}{24} \gamma^3 \hbar^3 \mathbf{B}_j^3 \mathbf{y}_j \mathbf{y}_k^3 \eta_i^4 + \frac{1}{12} \gamma^3 \hbar^3 \mathbf{B}_j^2 \mathbf{y}_j^2 \mathbf{y}_k^3 \eta_i^5 + \frac{1}{12} \gamma^3 \hbar^3 \mathbf{B}_j^3 \mathbf{y}_j^3 \mathbf{y}_k^3 \eta_i^5 + \\
 & \quad \frac{1}{48} \gamma^3 \hbar^3 \mathbf{B}_j^3 \mathbf{y}_j^3 \mathbf{y}_k^3 \eta_i^6 - \frac{1}{6} \hbar^3 \mathbf{a}_j^3 \mathbf{x}_k \xi_i - \frac{1}{4} \gamma^2 \hbar^3 \mathbf{a}_j \mathbf{B}_j \mathbf{x}_k \mathbf{y}_j \mathbf{y}_k \eta_i^2 \xi_i + \frac{1}{4} \gamma \hbar^3 \mathbf{a}_j^2 \mathbf{B}_j \mathbf{x}_k \mathbf{y}_j \mathbf{y}_k \eta_i^2 \xi_i - \\
 & \quad \frac{1}{6} \gamma^2 \hbar^3 \mathbf{a}_j \mathbf{B}_j \mathbf{x}_k \mathbf{y}_j^2 \mathbf{y}_k \eta_i^3 \xi_i - \frac{1}{6} \gamma^2 \hbar^3 \mathbf{a}_j \mathbf{B}_j^2 \mathbf{x}_k \mathbf{y}_j \mathbf{y}_k^2 \eta_i^3 \xi_i - \frac{1}{8} \gamma^2 \hbar^3 \mathbf{a}_j \mathbf{B}_j^2 \mathbf{x}_k \mathbf{y}_j^2 \mathbf{y}_k^2 \eta_i^4 \xi_i + \\
 & \quad \frac{1}{12} \gamma^3 \hbar^3 \mathbf{x}_j \mathbf{x}_k \xi_i^2 - \frac{1}{4} \gamma^2 \hbar^3 \mathbf{a}_j \mathbf{x}_j \mathbf{x}_k \xi_i^2 + \frac{1}{4} \gamma \hbar^3 \mathbf{a}_j^2 \mathbf{x}_j \mathbf{x}_k \xi_i^2 - \frac{1}{2} \hbar^3 \mathbf{a}_j^3 \mathbf{x}_k^2 \xi_i^2 + \\
 & \quad \frac{1}{4} \gamma^3 \hbar^3 \mathbf{B}_j \mathbf{x}_j \mathbf{x}_k \mathbf{y}_j \mathbf{y}_k \eta_i^2 \xi_i^2 - \frac{1}{4} \gamma^2 \hbar^3 \mathbf{a}_j \mathbf{B}_j \mathbf{x}_j \mathbf{x}_k \mathbf{y}_j \mathbf{y}_k \eta_i^2 \xi_i^2 + \frac{1}{4} \gamma \hbar^3 \mathbf{a}_j^2 \mathbf{B}_j \mathbf{x}_k^2 \mathbf{y}_j \mathbf{y}_k \eta_i^2 \xi_i^2 + \\
 & \quad \frac{1}{12} \gamma^3 \hbar^3 \mathbf{B}_j \mathbf{x}_j \mathbf{x}_k \mathbf{y}_j^2 \mathbf{y}_k \eta_i^3 \xi_i^2 + \frac{1}{12} \gamma^3 \hbar^3 \mathbf{B}_j^2 \mathbf{x}_j \mathbf{x}_k \mathbf{y}_j \mathbf{y}_k^2 \eta_i^3 \xi_i^2 + \frac{1}{16} \gamma^3 \hbar^3 \mathbf{B}_j^2 \mathbf{x}_j \mathbf{x}_k \mathbf{y}_j^2 \mathbf{y}_k^2 \eta_i^4 \xi_i^2 + \\
 & \quad \frac{1}{6} \gamma^3 \hbar^3 \mathbf{x}_j^2 \mathbf{x}_k \xi_i^3 - \frac{1}{6} \gamma^2 \hbar^3 \mathbf{a}_j \mathbf{x}_j^2 \mathbf{x}_k \xi_i^3 + \frac{1}{6} \gamma^3 \hbar^3 \mathbf{x}_j \mathbf{x}_k^2 \xi_i^3 - \frac{7}{12} \gamma^2 \hbar^3 \mathbf{a}_j \mathbf{x}_j \mathbf{x}_k^2 \xi_i^3 + \\
 & \quad \frac{3}{4} \gamma \hbar^3 \mathbf{a}_j^2 \mathbf{x}_j \mathbf{x}_k^2 \xi_i^3 - \frac{1}{6} \hbar^3 \mathbf{a}_j^3 \mathbf{x}_k^3 \xi_i^3 + \frac{1}{12} \gamma^3 \hbar^3 \mathbf{B}_j \mathbf{x}_j^2 \mathbf{x}_k \mathbf{y}_j \mathbf{y}_k \eta_i^3 \xi_i^3 + \frac{1}{12} \gamma^3 \hbar^3 \mathbf{B}_j \mathbf{x}_j \mathbf{x}_k^2 \mathbf{y}_j \mathbf{y}_k \eta_i^3 \xi_i^3 - \\
 & \quad \frac{1}{4} \gamma^2 \hbar^3 \mathbf{a}_j \mathbf{B}_j \mathbf{x}_j \mathbf{x}_k^2 \mathbf{y}_j \mathbf{y}_k \eta_i^3 \xi_i^3 + \frac{1}{24} \gamma^3 \hbar^3 \mathbf{x}_j^3 \mathbf{x}_k \xi_i^4 + \frac{1}{3} \gamma^3 \hbar^3 \mathbf{x}_j^2 \mathbf{x}_k^2 \xi_i^4 - \frac{5}{12} \gamma^2 \hbar^3 \mathbf{a}_j \mathbf{x}_j^2 \mathbf{x}_k^2 \xi_i^4 + \\
 & \quad \frac{1}{24} \gamma^3 \hbar^3 \mathbf{x}_j \mathbf{x}_k^3 \xi_i^4 - \frac{1}{6} \gamma^2 \hbar^3 \mathbf{a}_j \mathbf{x}_j \mathbf{x}_k^3 \xi_i^4 + \frac{1}{4} \gamma \hbar^3 \mathbf{a}_j^2 \mathbf{x}_j \mathbf{x}_k^3 \xi_i^4 + \frac{1}{16} \gamma^3 \hbar^3 \mathbf{B}_j \mathbf{x}_j^2 \mathbf{x}_k^2 \mathbf{y}_j \mathbf{y}_k \eta_i^2 \xi_i^4 + \\
 & \quad \left. \frac{1}{12} \gamma^3 \hbar^3 \mathbf{x}_j^3 \mathbf{x}_k^2 \xi_i^5 + \frac{1}{12} \gamma^3 \hbar^3 \mathbf{x}_j^2 \mathbf{x}_k^3 \xi_i^5 - \frac{1}{8} \gamma^2 \hbar^3 \mathbf{a}_j \mathbf{x}_j^2 \mathbf{x}_k^3 \xi_i^5 + \frac{1}{48} \gamma^3 \hbar^3 \mathbf{x}_j^3 \mathbf{x}_k^3 \xi_i^6 \right) \epsilon^3 + \mathbf{O}[\epsilon^4]
 \end{aligned}$$

$$\mathbf{C} \rightarrow \mathbb{E}_{\{\} \rightarrow \{i\}} \left[ \mathbf{0}, \mathbf{0}, \sqrt{\mathbf{B}_i} - \frac{1}{2} \left( \hbar \mathbf{a}_i \sqrt{\mathbf{B}_i} \right) \epsilon + \frac{1}{8} \hbar^2 \mathbf{a}_i^2 \sqrt{\mathbf{B}_i} \epsilon^2 - \frac{1}{48} \left( \hbar^3 \mathbf{a}_i^3 \sqrt{\mathbf{B}_i} \right) \epsilon^3 + \mathbf{O}[\epsilon^4] \right]$$

$$\bar{\mathbf{C}} \rightarrow \mathbb{E}_{\{\} \rightarrow \{i\}} \left[ \mathbf{0}, \mathbf{0}, \frac{1}{\sqrt{\mathbf{B}_i}} + \frac{\hbar \mathbf{a}_i \epsilon}{2\sqrt{\mathbf{B}_i}} + \frac{\hbar^2 \mathbf{a}_i^2 \epsilon^2}{8\sqrt{\mathbf{B}_i}} + \frac{\hbar^3 \mathbf{a}_i^3 \epsilon^3}{48\sqrt{\mathbf{B}_i}} + \mathbf{O}[\epsilon^4] \right]$$

Kink  $\rightarrow$

$$\begin{aligned}
 \mathbb{E}_{\{\} \rightarrow \{i\}} & \left[ \hbar \mathbf{a}_i \mathbf{b}_i, \hbar \mathbf{x}_i \mathbf{y}_i, \frac{1}{\sqrt{\mathbf{B}_i}} + \left( \frac{\hbar \mathbf{a}_i}{2\sqrt{\mathbf{B}_i}} - \frac{\gamma \hbar^3 \mathbf{x}_i^2 \mathbf{y}_i^2}{4\sqrt{\mathbf{B}_i}} \right) \epsilon + \left( \frac{\hbar^2 \mathbf{a}_i^2}{8\sqrt{\mathbf{B}_i}} - \frac{\gamma \hbar^4 \mathbf{a}_i \mathbf{x}_i^2 \mathbf{y}_i^2}{8\sqrt{\mathbf{B}_i}} + \frac{\gamma^2 \hbar^5 \mathbf{x}_i^3 \mathbf{y}_i^3}{9\sqrt{\mathbf{B}_i}} + \frac{\gamma^2 \hbar^6 \mathbf{x}_i^4 \mathbf{y}_i^4}{32\sqrt{\mathbf{B}_i}} \right) \epsilon^2 + \right. \\
 & \left( \frac{\hbar^3 \mathbf{a}_i^3}{48\sqrt{\mathbf{B}_i}} + \frac{\gamma^3 \hbar^5 \mathbf{x}_i^2 \mathbf{y}_i^2}{48\sqrt{\mathbf{B}_i}} - \frac{\gamma \hbar^5 \mathbf{a}_i^2 \mathbf{x}_i^2 \mathbf{y}_i^2}{32\sqrt{\mathbf{B}_i}} + \frac{\gamma^2 \hbar^6 \mathbf{a}_i \mathbf{x}_i^3 \mathbf{y}_i^3}{18\sqrt{\mathbf{B}_i}} - \frac{\gamma^3 \hbar^7 \mathbf{x}_i^4 \mathbf{y}_i^4}{16\sqrt{\mathbf{B}_i}} + \frac{\gamma^2 \hbar^7 \mathbf{a}_i \mathbf{x}_i^4 \mathbf{y}_i^4}{64\sqrt{\mathbf{B}_i}} - \frac{\gamma^3 \hbar^8 \mathbf{x}_i^5 \mathbf{y}_i^5}{36\sqrt{\mathbf{B}_i}} - \frac{\gamma^3 \hbar^9 \mathbf{x}_i^6 \mathbf{y}_i^6}{384\sqrt{\mathbf{B}_i}} \right) \epsilon^3 + \mathbf{O}[\epsilon^4]
 \end{aligned}$$

$$\overline{\text{Kink}} \rightarrow \mathbb{E}_{\{i\} \rightarrow \{i\}} \left[ -\hbar a_i b_i, -\frac{\hbar x_i y_i}{B_i}, \sqrt{B_i} + \left( -\frac{1}{2} \hbar a_i \sqrt{B_i} - \frac{\hbar^2 a_i x_i y_i}{\sqrt{B_i}} - \frac{3 \gamma \hbar^3 x_i^2 y_i^2}{4 B_i^{3/2}} \right) \epsilon + \right. \\ \left. \left( \frac{1}{8} \hbar^2 a_i^2 \sqrt{B_i} + \frac{\gamma^2 \hbar^4 x_i^2 y_i^2}{2 B_i^{3/2}} - \frac{9 \gamma \hbar^4 a_i x_i y_i^2}{8 B_i^{3/2}} + \frac{\hbar^4 a_i^2 x_i^2 y_i^2}{2 B_i^{3/2}} - \frac{10 \gamma^2 \hbar^5 x_i^3 y_i^3}{9 B_i^{5/2}} + \frac{3 \gamma \hbar^5 a_i x_i^3 y_i^3}{4 B_i^{5/2}} + \frac{9 \gamma^2 \hbar^6 x_i^4 y_i^4}{32 B_i^{7/2}} \right) \epsilon^2 + \right. \\ \left. \left( -\frac{1}{48} \hbar^3 a_i^3 \sqrt{B_i} - \frac{\hbar^4 a_i^3 x_i y_i}{24 \sqrt{B_i}} - \frac{3 \gamma^3 \hbar^5 x_i^2 y_i^2}{16 B_i^{3/2}} + \frac{3 \gamma^2 \hbar^5 a_i x_i^2 y_i^2}{4 B_i^{3/2}} - \frac{27 \gamma \hbar^5 a_i^2 x_i^2 y_i^2}{32 B_i^{3/2}} + \right. \right. \\ \left. \frac{\hbar^5 a_i^3 x_i^2 y_i^2}{4 B_i^{3/2}} + \frac{2 \gamma^3 \hbar^6 x_i^3 y_i^3}{B_i^{5/2}} - \frac{59 \gamma^2 \hbar^6 a_i x_i^3 y_i^3}{18 B_i^{5/2}} + \frac{3 \gamma \hbar^6 a_i^2 x_i^3 y_i^3}{2 B_i^{5/2}} - \frac{\hbar^6 a_i^3 x_i^3 y_i^3}{6 B_i^{5/2}} - \frac{41 \gamma^3 \hbar^7 x_i^4 y_i^4}{16 B_i^{7/2}} + \right. \\ \left. \frac{1207 \gamma^2 \hbar^7 a_i x_i^4 y_i^4}{576 B_i^{7/2}} - \frac{3 \gamma \hbar^7 a_i^2 x_i^4 y_i^4}{8 B_i^{7/2}} + \frac{5 \gamma^3 \hbar^8 x_i^5 y_i^5}{6 B_i^{9/2}} - \frac{9 \gamma^2 \hbar^8 a_i x_i^5 y_i^5}{32 B_i^{9/2}} - \frac{9 \gamma^3 \hbar^9 x_i^6 y_i^6}{128 B_i^{11/2}} \right) \epsilon^3 + \mathcal{O}[\epsilon]^4 \right]$$

$$\text{b2t} \rightarrow \mathbb{E}_{\{i\} \rightarrow \{i\}} \left[ a_i \alpha_i - \frac{t_i \beta_i}{\gamma}, y_i \eta_i + x_i \xi_i, 1 + \frac{a_i \beta_i \epsilon}{\gamma} + \frac{a_i^2 \beta_i^2 \epsilon^2}{2 \gamma^2} + \frac{a_i^3 \beta_i^3 \epsilon^3}{6 \gamma^3} + \mathcal{O}[\epsilon]^4 \right]$$

$$\text{t2b} \rightarrow \mathbb{E}_{\{i\} \rightarrow \{i\}} \left[ a_i \alpha_i - \gamma b_i \tau_i, y_i \eta_i + x_i \xi_i, 1 + a_i \tau_i \epsilon + \frac{1}{2} a_i^2 \tau_i^2 \epsilon^2 + \frac{1}{6} a_i^3 \tau_i^3 \epsilon^3 + \mathcal{O}[\epsilon]^4 \right]$$

```
In[6]:= Print[degrule = Thread[{a, b, α, β, ξ, η, x, y, ħ, γ, ε, t, τ, T, B, A} →
  {1, 1, -1, -1, -1, 1, 1, -2, 1, 1, 2, -2, 0, 0, 0}]];
atoms /. E_sp__[L_, Q_, P_] := (E[L, Q, P] ≡
  (E[L, Q, Normal@P] /. {v_i_ := s^v/.degrule v_i, (v : ħ | ε | γ) := s^v/.degrule v}))
{a → 1, b → 1, α → -1, β → -1, ξ → -1, η → -1, x → 1,
  y → 1, ħ → -2, γ → 1, ε → 1, t → 2, τ → -2, T → 0, B → 0, A → 0}
Out[6]:= {am → True, bm → True, dm → True, R → True, R̄ → True, P → True, aS → True,
  aS̄ → True, bS → True, bS̄ → True, dS → True, aΔ → True, bΔ → True, dΔ → True,
  C → True, C̄ → True, Kink → True, Kink̄ → True, b2t → True, t2b → True}
```

```
In[6]:= Print[degrule = Thread[{a, b, α, β, ξ, η, x, y, ħ, γ, ε, t, τ, T, B, A} →
  {0, 1, 0, -1, 0, -1, 0, 1, -1, 0, 1, 1, -1, 0, 0, 0}]];
atoms /. E_sp__[L_, Q_, P_] := (E[L, Q, P] ≡
  (E[L, Q, Normal@P] /. {v_i_ := s^v/.degrule v_i, (v : ħ | ε | γ) := s^v/.degrule v}))
{a → 0, b → 1, α → 0, β → -1, ξ → 0, η → -1, x → 0,
  y → 1, ħ → -1, γ → 0, ε → 1, t → 1, τ → -1, T → 0, B → 0, A → 0}
Out[6]:= {am → True, bm → True, dm → True, R → True, R̄ → True, P → True, aS → True,
  aS̄ → True, bS → True, bS̄ → True, dS → True, aΔ → True, bΔ → True, dΔ → True,
  C → True, C̄ → True, Kink → True, Kink̄ → True, b2t → True, t2b → True}
```

```
In[6]:= Print[degrule = Thread[{a, b, α, β, ξ, η, x, y, ħ, γ, ε, t, τ, T, B, A} →
  {1, 0, -1, 0, -1, 0, 1, 0, -1, 1, 0, 1, -1, 0, 0, 0}]];
atoms /. E_sp__[L_, Q_, P_] := (E[L, Q, P] ≡
  (E[L, Q, Normal@P] /. {v_i_ := s^v/.degrule v_i, (v : ħ | ε | γ) := s^v/.degrule v}))
{a → 1, b → 0, α → -1, β → 0, ξ → -1, η → 0, x → 1,
  y → 0, ħ → -1, γ → 1, ε → 0, t → 1, τ → -1, T → 0, B → 0, A → 0}
Out[6]:= {am → True, bm → True, dm → True, R → True, R̄ → True, P → True, aS → True,
  aS̄ → True, bS → True, bS̄ → True, dS → True, aΔ → True, bΔ → True, dΔ → True,
  C → True, C̄ → True, Kink → True, Kink̄ → True, b2t → True, t2b → True}
```

```
In[6]:= Column[atoms /. E_sp__[L_, Q_, P_] := EE_sp[L, Q, CF@Log[P]]]
am → EE_{i,j} → {k} [a_k (α_i + α_j), x_k (e^{-γ α_j} ξ_i + ξ_j), 0]
bm → EE_{i,j} → {k} [b_k (β_i + β_j), y_k (η_i + η_j), -y_k β_i η_j ε + 1/2 y_k β_i^2 η_j ε^2 - 1/6 (y_k β_i^3 η_j) ε^3 + O[ε]^4]
```

$$\begin{aligned}
 dm \rightarrow EE_{\{i,j\} \rightarrow \{k\}} & \left[ a_k \alpha_i + a_k \alpha_j + b_k \beta_i + b_k \beta_j, y_k \eta_i + \frac{y_k \eta_j}{\sigma_i} + \frac{x_k \epsilon_i}{\sigma_j} + \frac{(1-B_k) \eta_j \epsilon_i}{h} + X_k \epsilon_j, \right. \\
 & \left( -\frac{y_k \beta_i \eta_j}{\sigma_i} - \frac{x_k \beta_j \epsilon_i}{\sigma_j} + a_k B_k \eta_j \epsilon_i + \frac{\gamma h x_k y_k \eta_j \epsilon_i}{\sigma_i \sigma_j} + \frac{(\gamma-3 \gamma B_k) y_k \eta_j^2 \epsilon_i}{2 \sigma_i} + \frac{(\gamma-3 \gamma B_k) x_k \eta_j \epsilon_i^2}{2 \sigma_j} + \frac{(\gamma-4 \gamma B_k + 3 \gamma B_k^2) \eta_j^2 \epsilon_i^2}{4 h} \right) \in + \\
 & \left( \frac{y_k \beta_i^2 \eta_j}{2 \sigma_i} + \frac{x_k \beta_j^2 \epsilon_i}{2 \sigma_j} - \frac{1}{2} h a_k^2 B_k \eta_j \epsilon_i + \frac{\gamma^2 h^2 x_k y_k \eta_j \epsilon_i}{2 \sigma_i \sigma_j} - \frac{\gamma h x_k y_k \beta_i \eta_j \epsilon_i}{\sigma_i \sigma_j} - \frac{\gamma h x_k y_k \beta_j \eta_j \epsilon_i}{\sigma_i \sigma_j} + \frac{3 \gamma h a_k B_k y_k \eta_j \epsilon_i}{2 \sigma_i} + \right. \\
 & \frac{(\gamma^2 h - 5 \gamma^2 h B_k) y_k \eta_j^2 \epsilon_i}{4 \sigma_i} + \frac{\gamma^2 h^2 x_k y_k \eta_j^2 \epsilon_i}{2 \sigma_i^2 \sigma_j} + \frac{(-\gamma + 3 \gamma B_k) y_k \beta_i \eta_j^2 \epsilon_i}{2 \sigma_i} + \frac{(\gamma^2 h - 7 \gamma^2 h B_k) y_k^2 \eta_j^2 \epsilon_i}{6 \sigma_i^2} + \frac{\gamma^2 h^2 x_k^2 y_k \eta_j \epsilon_i^2}{2 \sigma_i \sigma_j^2} + \\
 & \frac{3 \gamma h a_k B_k x_k \eta_j \epsilon_i^2}{2 \sigma_j} + \frac{(\gamma^2 h - 5 \gamma^2 h B_k) x_k \eta_j \epsilon_i^2}{4 \sigma_j} + \frac{(-\gamma + 3 \gamma B_k) x_k \beta_j \eta_j \epsilon_i^2}{2 \sigma_j} + \frac{1}{2} a_k (2 \gamma B_k - 3 \gamma B_k^2) \eta_j^2 \epsilon_i^2 + \\
 & \frac{1}{8} (\gamma^2 - 6 \gamma^2 B_k + 5 \gamma^2 B_k^2) \eta_j^2 \epsilon_i^2 + \frac{(5 \gamma^2 h - 21 \gamma^2 h B_k) x_k y_k \eta_j^2 \epsilon_i^2}{4 \sigma_i \sigma_j} + \frac{(5 \gamma^2 - 34 \gamma^2 B_k + 41 \gamma^2 B_k^2) y_k \eta_j^2 \epsilon_i^2}{12 \sigma_i} + \\
 & \left. \frac{(\gamma^2 h - 7 \gamma^2 h B_k) x_k^2 \eta_j \epsilon_i^2}{6 \sigma_j^2} + \frac{(5 \gamma^2 - 34 \gamma^2 B_k + 41 \gamma^2 B_k^2) x_k \eta_j^2 \epsilon_i^2}{12 \sigma_j} + \frac{(5 \gamma^2 - 39 \gamma^2 B_k + 75 \gamma^2 B_k^2 - 41 \gamma^2 B_k^3) \eta_j^3 \epsilon_i^2}{36 h} \right) \in 2 + \\
 & \left( -\frac{y_k \beta_i^2 \eta_j}{6 \sigma_i} - \frac{x_k \beta_j^2 \epsilon_i}{6 \sigma_j} + \frac{1}{6} h^2 a_k^3 B_k \eta_j \epsilon_i + \frac{\gamma^3 h^3 x_k y_k \eta_j \epsilon_i}{6 \sigma_i \sigma_j} - \frac{\gamma^2 h^2 x_k y_k \beta_i \eta_j \epsilon_i}{2 \sigma_i \sigma_j} + \frac{\gamma h x_k y_k \beta_j^2 \eta_j \epsilon_i}{2 \sigma_i \sigma_j} - \right. \\
 & \frac{\gamma^2 h^2 x_k y_k \beta_j \eta_j \epsilon_i}{2 \sigma_i \sigma_j} + \frac{\gamma h x_k y_k \beta_i \beta_j \eta_j \epsilon_i}{\sigma_i \sigma_j} + \frac{\gamma h x_k y_k \beta_j^2 \eta_j \epsilon_i}{2 \sigma_i \sigma_j} + \frac{5 \gamma^2 h^2 a_k B_k y_k \eta_j^2 \epsilon_i}{4 \sigma_i} - \frac{3 \gamma h^2 a_k^2 B_k y_k \eta_j^2 \epsilon_i}{4 \sigma_i} + \\
 & \frac{(\gamma^3 h^2 - 9 \gamma^3 h^2 B_k) y_k \eta_j^2 \epsilon_i}{12 \sigma_i} + \frac{\gamma^3 h^3 x_k y_k^2 \eta_j^2 \epsilon_i}{2 \sigma_i^2 \sigma_j} - \frac{3 \gamma h a_k B_k y_k \beta_i \eta_j^2 \epsilon_i}{2 \sigma_i} + \frac{(-\gamma^2 h + 5 \gamma^2 h B_k) y_k \beta_i \eta_j^2 \epsilon_i}{4 \sigma_i} - \\
 & \frac{\gamma^2 h^2 x_k y_k^2 \beta_i \eta_j^2 \epsilon_i}{\sigma_i^2 \sigma_j} + \frac{(\gamma - 3 \gamma B_k) y_k \beta_j^2 \eta_j^2 \epsilon_i}{4 \sigma_i} - \frac{\gamma^2 h^2 x_k y_k \beta_j \eta_j^2 \epsilon_i}{2 \sigma_i^2 \sigma_j} + \frac{7 \gamma^2 h^2 a_k B_k y_k^2 \eta_j^2 \epsilon_i}{6 \sigma_i^2} + \frac{(\gamma^3 h^2 - 12 \gamma^3 h^2 B_k) y_k^2 \eta_j^2 \epsilon_i}{6 \sigma_i^2} + \\
 & \frac{\gamma^3 h^3 x_k y_k^2 \eta_j^2 \epsilon_i}{6 \sigma_i^2 \sigma_j} + \frac{(-\gamma^2 h + 7 \gamma^2 h B_k) y_k^2 \beta_i \eta_j^2 \epsilon_i}{3 \sigma_i^2} + \frac{(\gamma^3 h^2 - 15 \gamma^3 h^2 B_k) y_k^2 \eta_j^4 \epsilon_i}{24 \sigma_i^3} + \frac{\gamma^3 h^3 x_k^2 y_k \eta_j \epsilon_i^2}{2 \sigma_i \sigma_j^2} + \\
 & \frac{5 \gamma^2 h^2 a_k B_k x_k \eta_j \epsilon_i^2}{4 \sigma_j} - \frac{3 \gamma h^2 a_k^2 B_k x_k \eta_j \epsilon_i^2}{4 \sigma_j} + \frac{(\gamma^3 h^2 - 9 \gamma^3 h^2 B_k) x_k \eta_j \epsilon_i^2}{12 \sigma_j} - \frac{\gamma^2 h^2 x_k^2 y_k \beta_i \eta_j \epsilon_i^2}{2 \sigma_i \sigma_j^2} - \frac{\gamma^2 h^2 x_k^2 y_k \beta_j \eta_j \epsilon_i^2}{\sigma_i \sigma_j^2} - \\
 & \frac{3 \gamma h a_k B_k x_k \beta_j \eta_j \epsilon_i^2}{2 \sigma_j} + \frac{(-\gamma^2 h + 5 \gamma^2 h B_k) x_k \beta_j \eta_j \epsilon_i^2}{4 \sigma_j} + \frac{(\gamma - 3 \gamma B_k) x_k \beta_j^2 \eta_j \epsilon_i^2}{4 \sigma_j} + \frac{1}{2} a_k^2 (-\gamma h B_k + 3 \gamma h B_k^2) \eta_j^2 \epsilon_i^2 + \\
 & \frac{1}{4} a_k (3 \gamma^2 h B_k - 5 \gamma^2 h B_k^2) \eta_j^2 \epsilon_i^2 + \frac{1}{24} (\gamma^3 h - 10 \gamma^3 h B_k + 9 \gamma^3 h B_k^2) \eta_j^2 \epsilon_i^2 + \\
 & \frac{\gamma^3 h^3 x_k^2 y_k^2 \eta_j^2 \epsilon_i^2}{\sigma_i^2 \sigma_j^2} + \frac{21 \gamma^2 h^2 a_k B_k x_k y_k \eta_j^2 \epsilon_i^2}{4 \sigma_i \sigma_j} + \frac{(3 \gamma^3 h^2 - 20 \gamma^3 h^2 B_k) x_k y_k \eta_j^2 \epsilon_i^2}{2 \sigma_i \sigma_j} + \frac{(-5 \gamma^2 h + 21 \gamma^2 h B_k) x_k y_k \beta_i \eta_j^2 \epsilon_i^2}{4 \sigma_i \sigma_j} + \\
 & \frac{(-5 \gamma^2 h + 21 \gamma^2 h B_k) x_k y_k \beta_j \eta_j^2 \epsilon_i^2}{4 \sigma_i \sigma_j} + \frac{a_k (17 \gamma^2 h B_k - 41 \gamma^2 h B_k^2) y_k \eta_j^2 \epsilon_i^2}{6 \sigma_i} + \frac{(\gamma^3 h - 10 \gamma^3 h B_k + 15 \gamma^3 h B_k^2) y_k \eta_j^2 \epsilon_i^2}{2 \sigma_i} + \\
 & \frac{(17 \gamma^3 h^2 - 103 \gamma^3 h^2 B_k) x_k y_k^2 \eta_j^2 \epsilon_i^2}{12 \sigma_i^2 \sigma_j} + \frac{(-5 \gamma^2 + 34 \gamma^2 B_k - 41 \gamma^2 B_k^2) y_k \beta_i \eta_j^2 \epsilon_i^2}{12 \sigma_i} + \frac{(17 \gamma^3 h - 188 \gamma^3 h B_k + 339 \gamma^3 h B_k^2) y_k^2 \eta_j^4 \epsilon_i^2}{48 \sigma_i^2} + \\
 & \frac{\gamma^3 h^3 x_k^3 y_k \eta_j \epsilon_i^2}{6 \sigma_i \sigma_j^2} + \frac{7 \gamma^2 h^2 a_k B_k x_k^2 \eta_j \epsilon_i^2}{6 \sigma_j^2} + \frac{(\gamma^3 h^2 - 12 \gamma^3 h^2 B_k) x_k^2 \eta_j \epsilon_i^2}{6 \sigma_j^2} + \frac{(-\gamma^2 h + 7 \gamma^2 h B_k) x_k^2 \beta_j \eta_j \epsilon_i^2}{3 \sigma_j^2} + \\
 & \frac{(17 \gamma^3 h^2 - 103 \gamma^3 h^2 B_k) x_k^2 y_k \eta_j^2 \epsilon_i^2}{12 \sigma_i \sigma_j^2} + \frac{a_k (17 \gamma^2 h B_k - 41 \gamma^2 h B_k^2) x_k \eta_j^2 \epsilon_i^2}{6 \sigma_j} + \frac{(\gamma^3 h - 10 \gamma^3 h B_k + 15 \gamma^3 h B_k^2) x_k \eta_j^2 \epsilon_i^2}{2 \sigma_j} + \\
 & \frac{(-5 \gamma^2 + 34 \gamma^2 B_k - 41 \gamma^2 B_k^2) x_k \beta_j \eta_j^2 \epsilon_i^2}{12 \sigma_j} + \frac{1}{12} a_k (13 \gamma^2 B_k - 50 \gamma^2 B_k^2 + 41 \gamma^2 B_k^3) \eta_j^2 \epsilon_i^2 + \\
 & \frac{1}{6} (\gamma^3 - 11 \gamma^3 B_k + 25 \gamma^3 B_k^2 - 15 \gamma^3 B_k^3) \eta_j^2 \epsilon_i^2 + \frac{(10 \gamma^3 h - 87 \gamma^3 h B_k + 137 \gamma^3 h B_k^2) x_k y_k \eta_j^2 \epsilon_i^2}{6 \sigma_i \sigma_j} + \\
 & \frac{(5 \gamma^3 - 55 \gamma^3 B_k + 149 \gamma^3 B_k^2 - 111 \gamma^3 B_k^3) y_k \eta_j^4 \epsilon_i^2}{12 \sigma_i} + \frac{(\gamma^3 h^2 - 15 \gamma^3 h^2 B_k) x_k^2 \eta_j \epsilon_i^2}{24 \sigma_j^2} + \frac{(17 \gamma^3 h - 188 \gamma^3 h B_k + 339 \gamma^3 h B_k^2) x_k^2 \eta_j^2 \epsilon_i^2}{48 \sigma_j^2} + \\
 & \left. \frac{(5 \gamma^3 - 55 \gamma^3 B_k + 149 \gamma^3 B_k^2 - 111 \gamma^3 B_k^3) x_k \eta_j^3 \epsilon_i^2}{12 \sigma_j} + \frac{(5 \gamma^3 - 60 \gamma^3 B_k + 204 \gamma^3 B_k^2 - 260 \gamma^3 B_k^3 + 111 \gamma^3 B_k^4) \eta_j^4 \epsilon_i^2}{48 h} \right) \in 3 + O[\epsilon]^4
 \end{aligned}$$

$$\begin{aligned}
 R \rightarrow EE_{\{\} \rightarrow \{i,j\}} & \left[ h a_j b_i, h x_j y_i, \right. \\
 & \left. - \frac{1}{4} (\gamma h^3 x_j^2 y_i^2) \in + \frac{1}{9} \gamma^2 h^5 x_j^3 y_i^2 \in 2 + \left( \frac{1}{48} \gamma^3 h^5 x_j^2 y_i^2 - \frac{1}{16} \gamma^3 h^7 x_j^4 y_i^4 \right) \in 3 + O[\epsilon]^4 \right]
 \end{aligned}$$

$$\begin{aligned}
 \bar{R} \rightarrow EE_{\{\} \rightarrow \{i,j\}} & \left[ -h a_j b_i, -\frac{h x_j y_i}{B_i}, \right. \\
 & \left( -\frac{h^2 a_j x_j y_i}{B_i} - \frac{3 \gamma h^3 x_j^2 y_i^2}{4 B_i^2} \right) \in + \left( -\frac{h^3 a_j^2 x_j y_i}{2 B_i} + \frac{\gamma^2 h^4 x_j^2 y_i^2}{2 B_i^2} - \frac{3 \gamma h^4 a_j x_j^2 y_i^2}{2 B_i^2} - \frac{10 \gamma^2 h^5 x_j^3 y_i^3}{9 B_i^2} \right) \in 2 + \\
 & \left( -\frac{h^4 a_j^3 x_j y_i}{6 B_i} - \frac{3 \gamma h^5 x_j^2 y_i^2}{16 B_i^2} + \frac{\gamma^2 h^5 a_j x_j^2 y_i^2}{B_i^2} - \frac{3 \gamma h^5 a_j^2 x_j^2 y_i^2}{2 B_i^2} + \frac{2 \gamma^3 h^6 x_j^3 y_i^3}{B_i^3} - \frac{10 \gamma^2 h^6 a_j x_j^3 y_i^3}{3 B_i^3} - \frac{35 \gamma^3 h^7 x_j^4 y_i^4}{16 B_i^4} \right) \in 3 + O[\epsilon]^4
 \end{aligned}$$

$$\begin{aligned}
 P \rightarrow EE_{\{i,j\} \rightarrow \{\}} & \left[ \frac{\alpha_j \beta_i}{h}, \frac{\eta_i \epsilon_j}{h}, \right. \\
 & \left. \frac{\gamma \eta_i^2 \epsilon_j^2}{4 h} \in + \left( \frac{1}{8} \gamma^2 \eta_i^2 \epsilon_j^2 + \frac{5 \gamma^2 \eta_i^2 \epsilon_j^3}{36 h} \right) \in 2 + \left( \frac{1}{24} \gamma^3 h \eta_i^2 \epsilon_j^2 + \frac{1}{6} \gamma^3 \eta_i^3 \epsilon_j^3 + \frac{5 \gamma^3 \eta_i^4 \epsilon_j^4}{48 h} \right) \in 3 + O[\epsilon]^4 \right]
 \end{aligned}$$

$$\begin{aligned} \mathbf{aS} \rightarrow \mathbb{E}\mathbb{E}_{\{i\} \rightarrow \{i\}} & \left[ -\mathbf{a}_i \alpha_i, -\mathbf{x}_i \mathcal{A}_i \xi_i, \left( -\hbar \mathbf{a}_i \mathbf{x}_i \mathcal{A}_i \xi_i - \frac{1}{2} \gamma \hbar \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^2 \right) \in + \right. \\ & \left( -\frac{1}{2} \hbar^2 \mathbf{a}_i^2 \mathbf{x}_i \mathcal{A}_i \xi_i + \frac{1}{4} \gamma^2 \hbar^2 \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^2 - \gamma \hbar^2 \mathbf{a}_i \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^2 - \frac{1}{2} \gamma^2 \hbar^2 \mathbf{x}_i^3 \mathcal{A}_i^3 \xi_i^3 \right) \in^2 + \\ & \left( -\frac{1}{6} \hbar^3 \mathbf{a}_i^3 \mathbf{x}_i \mathcal{A}_i \xi_i - \frac{1}{12} \gamma^3 \hbar^3 \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^2 + \frac{1}{2} \gamma^2 \hbar^3 \mathbf{a}_i \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^2 - \gamma \hbar^3 \mathbf{a}_i^2 \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^2 + \right. \\ & \left. \frac{2}{3} \gamma^3 \hbar^3 \mathbf{x}_i^3 \mathcal{A}_i^3 \xi_i^3 - \frac{2}{3} \gamma^2 \hbar^3 \mathbf{a}_i \mathbf{x}_i^3 \mathcal{A}_i^3 \xi_i^3 - \frac{2}{3} \gamma^3 \hbar^3 \mathbf{x}_i^4 \mathcal{A}_i^4 \xi_i^4 \right) \in^3 + \mathbf{O}[\epsilon]^4 \end{aligned}$$

$$\begin{aligned} \overline{\mathbf{aS}} \rightarrow \mathbb{E}\mathbb{E}_{\{i\} \rightarrow \{i\}} & \left[ -\mathbf{a}_i \alpha_i, -\mathbf{x}_i \mathcal{A}_i \xi_i, \right. \\ & \left( \gamma \hbar \mathbf{x}_i \mathcal{A}_i \xi_i - \hbar \mathbf{a}_i \mathbf{x}_i \mathcal{A}_i \xi_i - \frac{1}{2} \gamma \hbar \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^2 \right) \in + \left( -\frac{1}{2} \gamma^2 \hbar^2 \mathbf{x}_i \mathcal{A}_i \xi_i + \gamma \hbar^2 \mathbf{a}_i \mathbf{x}_i \mathcal{A}_i \xi_i - \right. \\ & \left. \frac{1}{2} \hbar^2 \mathbf{a}_i^2 \mathbf{x}_i \mathcal{A}_i \xi_i + \frac{5}{4} \gamma^2 \hbar^2 \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^2 - \gamma \hbar^2 \mathbf{a}_i \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^2 - \frac{1}{2} \gamma^2 \hbar^2 \mathbf{x}_i^3 \mathcal{A}_i^3 \xi_i^3 \right) \in^2 + \\ & \left( \frac{1}{6} \gamma^3 \hbar^3 \mathbf{x}_i \mathcal{A}_i \xi_i - \frac{1}{2} \gamma^2 \hbar^3 \mathbf{a}_i \mathbf{x}_i \mathcal{A}_i \xi_i + \frac{1}{2} \gamma \hbar^3 \mathbf{a}_i^2 \mathbf{x}_i \mathcal{A}_i \xi_i - \frac{1}{6} \hbar^3 \mathbf{a}_i^3 \mathbf{x}_i \mathcal{A}_i \xi_i - \frac{19}{12} \gamma^3 \hbar^3 \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^2 + \frac{5}{2} \gamma^2 \hbar^3 \right. \\ & \left. \mathbf{a}_i \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^2 - \gamma \hbar^3 \mathbf{a}_i^2 \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^2 + \frac{13}{6} \gamma^3 \hbar^3 \mathbf{x}_i^3 \mathcal{A}_i^3 \xi_i^3 - \frac{3}{2} \gamma^2 \hbar^3 \mathbf{a}_i \mathbf{x}_i^3 \mathcal{A}_i^3 \xi_i^3 - \frac{2}{3} \gamma^3 \hbar^3 \mathbf{x}_i^4 \mathcal{A}_i^4 \xi_i^4 \right) \in^3 + \mathbf{O}[\epsilon]^4 \end{aligned}$$

$$\begin{aligned} \mathbf{bS} \rightarrow \mathbb{E}\mathbb{E}_{\{i\} \rightarrow \{i\}} & \left[ -\mathbf{b}_i \beta_i, -\frac{\mathbf{y}_i \eta_i}{\mathbf{B}_i}, \left( -\frac{\mathbf{y}_i \beta_i \eta_i}{\mathbf{B}_i} - \frac{\gamma \hbar \mathbf{y}_i^2 \eta_i^2}{2 \mathbf{B}_i^2} \right) \in + \left( -\frac{\mathbf{y}_i \beta_i^2 \eta_i}{2 \mathbf{B}_i} + \frac{\gamma^2 \hbar^2 \mathbf{y}_i^2 \eta_i^2}{4 \mathbf{B}_i^2} - \frac{\gamma \hbar \mathbf{y}_i^2 \beta_i \eta_i}{\mathbf{B}_i^2} - \frac{\gamma^2 \hbar^2 \mathbf{y}_i^3 \eta_i^3}{2 \mathbf{B}_i^3} \right) \in^2 + \right. \\ & \left( -\frac{\mathbf{y}_i \beta_i^3 \eta_i}{6 \mathbf{B}_i} - \frac{\gamma^3 \hbar^3 \mathbf{y}_i^2 \eta_i^2}{12 \mathbf{B}_i^2} + \frac{\gamma^2 \hbar^2 \mathbf{y}_i^2 \beta_i \eta_i}{2 \mathbf{B}_i^2} - \frac{\gamma \hbar \mathbf{y}_i^2 \beta_i^2 \eta_i}{\mathbf{B}_i^2} + \frac{2 \gamma^3 \hbar^3 \mathbf{y}_i^3 \eta_i^3}{3 \mathbf{B}_i^3} - \frac{3 \gamma^2 \hbar^2 \mathbf{y}_i^3 \beta_i \eta_i}{2 \mathbf{B}_i^3} - \frac{2 \gamma^3 \hbar^3 \mathbf{y}_i^4 \eta_i^4}{3 \mathbf{B}_i^4} \right) \in^3 + \mathbf{O}[\epsilon]^4 \end{aligned}$$

$$\begin{aligned} \overline{\mathbf{bS}} \rightarrow \mathbb{E}\mathbb{E}_{\{i\} \rightarrow \{i\}} & \left[ -\mathbf{b}_i \beta_i, -\frac{\mathbf{y}_i \eta_i}{\mathbf{B}_i}, \right. \\ & \left( \frac{\gamma \hbar \mathbf{y}_i \eta_i}{\mathbf{B}_i} - \frac{\mathbf{y}_i \beta_i \eta_i}{\mathbf{B}_i} - \frac{\gamma \hbar \mathbf{y}_i^2 \eta_i^2}{2 \mathbf{B}_i^2} \right) \in + \left( -\frac{\gamma^2 \hbar^2 \mathbf{y}_i \eta_i}{2 \mathbf{B}_i} + \frac{\gamma \hbar \mathbf{y}_i \beta_i \eta_i}{\mathbf{B}_i} - \frac{\mathbf{y}_i \beta_i^2 \eta_i}{2 \mathbf{B}_i} + \frac{5 \gamma^2 \hbar^2 \mathbf{y}_i^2 \eta_i^2}{4 \mathbf{B}_i^2} - \frac{\gamma \hbar \mathbf{y}_i^2 \beta_i \eta_i}{\mathbf{B}_i^2} - \frac{\gamma^2 \hbar^2 \mathbf{y}_i^3 \eta_i^3}{2 \mathbf{B}_i^3} \right) \in^2 + \\ & \left( \frac{\gamma^3 \hbar^3 \mathbf{y}_i \eta_i}{6 \mathbf{B}_i} - \frac{\gamma^2 \hbar^2 \mathbf{y}_i \beta_i \eta_i}{2 \mathbf{B}_i} + \frac{\gamma \hbar \mathbf{y}_i \beta_i^2 \eta_i}{2 \mathbf{B}_i} - \frac{\mathbf{y}_i \beta_i^3 \eta_i}{6 \mathbf{B}_i} - \frac{19 \gamma^3 \hbar^3 \mathbf{y}_i^2 \eta_i^2}{12 \mathbf{B}_i^2} + \right. \\ & \left. \frac{5 \gamma^2 \hbar^2 \mathbf{y}_i^2 \beta_i \eta_i}{2 \mathbf{B}_i^2} - \frac{\gamma \hbar \mathbf{y}_i^2 \beta_i^2 \eta_i}{\mathbf{B}_i^2} + \frac{13 \gamma^3 \hbar^3 \mathbf{y}_i^3 \eta_i^3}{6 \mathbf{B}_i^3} - \frac{3 \gamma^2 \hbar^2 \mathbf{y}_i^3 \beta_i \eta_i}{2 \mathbf{B}_i^3} - \frac{2 \gamma^3 \hbar^3 \mathbf{y}_i^4 \eta_i^4}{3 \mathbf{B}_i^4} \right) \in^3 + \mathbf{O}[\epsilon]^4 \end{aligned}$$



$$\begin{aligned}
 dS \rightarrow EE_{\{i\} \rightarrow \{i\}} & \left[ -\mathbf{a}_i \alpha_i - \mathbf{b}_i \beta_i, -\frac{y_i \mathcal{A}_i \eta_i}{B_i} - \mathbf{x}_i \mathcal{A}_i \xi_i + \frac{(\mathcal{A}_i - B_i \mathcal{A}_i) \eta_i \xi_i}{h B_i}, \right. \\
 & \left( \frac{\gamma h y_i \mathcal{A}_i \eta_i}{B_i} - \frac{y_i \mathcal{A}_i \beta_i \eta_i}{B_i} - \frac{\gamma h y_i^2 \mathcal{A}_i^2 \eta_i^2}{2 B_i^2} - h \mathbf{a}_i \mathbf{x}_i \mathcal{A}_i \xi_i - \mathbf{x}_i \mathcal{A}_i \beta_i \xi_i + \frac{\mathbf{a}_i \mathcal{A}_i \eta_i \xi_i}{B_i} - \right. \\
 & \frac{\gamma h \mathbf{x}_i y_i \mathcal{A}_i \eta_i \xi_i}{B_i} + \frac{(-\gamma \mathcal{A}_i + \gamma B_i \mathcal{A}_i) \eta_i \xi_i}{B_i} + \frac{(\mathcal{A}_i - B_i \mathcal{A}_i) \beta_i \eta_i \xi_i}{h B_i} + \frac{y_i (3 \gamma \mathcal{A}_i^2 - \gamma B_i \mathcal{A}_i^2) \eta_i^2 \xi_i}{2 B_i^2} - \\
 & \left. \frac{1}{2} \gamma h \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^2 + \frac{\mathbf{x}_i (3 \gamma \mathcal{A}_i^2 - \gamma B_i \mathcal{A}_i^2) \eta_i \xi_i^2}{2 B_i} + \frac{(-3 \gamma \mathcal{A}_i^2 + 4 \gamma B_i \mathcal{A}_i^2 - \gamma B_i^2 \mathcal{A}_i^2) \eta_i^2 \xi_i^2}{4 h B_i^2} \right) \in + \\
 & \left( -\frac{\gamma^2 h^2 y_i \mathcal{A}_i \eta_i}{2 B_i} + \frac{\gamma h y_i \mathcal{A}_i \beta_i \eta_i}{B_i} - \frac{y_i \mathcal{A}_i \beta_i^2 \eta_i}{2 B_i} + \frac{5 \gamma^2 h^2 y_i^2 \mathcal{A}_i^2 \eta_i^2}{4 B_i^2} - \frac{\gamma h y_i^2 \mathcal{A}_i^2 \beta_i \eta_i^2}{B_i^2} - \frac{\gamma^2 h^2 y_i^3 \mathcal{A}_i^3 \eta_i^3}{2 B_i^2} - \frac{1}{2} h^2 \mathbf{a}_i^2 \mathbf{x}_i \mathcal{A}_i \xi_i - \right. \\
 & h \mathbf{a}_i \mathbf{x}_i \mathcal{A}_i \beta_i \xi_i - \frac{1}{2} \mathbf{x}_i \mathcal{A}_i \beta_i^2 \xi_i - \frac{\gamma h \mathbf{a}_i \mathcal{A}_i \eta_i \xi_i}{B_i} + \frac{h \mathbf{a}_i^2 \mathcal{A}_i \eta_i \xi_i}{2 B_i} + \frac{3 \gamma^2 h^2 \mathbf{x}_i y_i \mathcal{A}_i^2 \eta_i \xi_i}{2 B_i} - \frac{\gamma h^2 \mathbf{a}_i \mathbf{x}_i y_i \mathcal{A}_i^2 \eta_i \xi_i}{B_i} + \\
 & \frac{(\gamma^2 h \mathcal{A}_i - \gamma^2 h B_i \mathcal{A}_i) \eta_i \xi_i}{2 B_i} + \frac{\mathbf{a}_i \mathcal{A}_i \beta_i \eta_i \xi_i}{B_i} - \frac{2 \gamma h \mathbf{x}_i y_i \mathcal{A}_i^2 \beta_i \eta_i \xi_i}{B_i} + \frac{(-\gamma \mathcal{A}_i + \gamma B_i \mathcal{A}_i) \beta_i \eta_i \xi_i}{B_i} + \frac{(\mathcal{A}_i - B_i \mathcal{A}_i) \beta_i^2 \eta_i \xi_i}{2 h B_i} + \\
 & \frac{3 \gamma h \mathbf{a}_i y_i \mathcal{A}_i^2 \eta_i^2 \xi_i}{2 B_i^2} - \frac{3 \gamma^2 h^2 \mathbf{x}_i y_i^2 \mathcal{A}_i^2 \eta_i^2 \xi_i}{4 B_i^2} + \frac{y_i (-17 \gamma^2 h \mathcal{A}_i^2 + 5 \gamma^2 h B_i \mathcal{A}_i^2) \eta_i^2 \xi_i}{4 B_i^2} + \frac{y_i (3 \gamma \mathcal{A}_i^2 - \gamma B_i \mathcal{A}_i^2) \beta_i \eta_i^2 \xi_i}{B_i^2} + \\
 \text{Out[ ]=} & \frac{y_i^2 (8 \gamma^2 h^2 \mathcal{A}_i - 2 \gamma^2 h B_i \mathcal{A}_i) \eta_i^3 \xi_i}{3 B_i^2} + \frac{1}{4} \gamma^2 h^2 \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^2 - \gamma h^2 \mathbf{a}_i \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^2 - \gamma h \mathbf{x}_i^2 \mathcal{A}_i^2 \beta_i \xi_i^2 - \frac{3 \gamma^2 h^2 \mathbf{x}_i y_i \mathcal{A}_i^2 \eta_i \xi_i^2}{2 B_i} + \\
 & \frac{\mathbf{a}_i \mathbf{x}_i (6 \gamma h \mathcal{A}_i^2 - \gamma h B_i \mathcal{A}_i^2) \eta_i \xi_i^2}{2 B_i} + \frac{\mathbf{x}_i (-11 \gamma^2 h \mathcal{A}_i^2 + 3 \gamma^2 h B_i \mathcal{A}_i^2) \eta_i \xi_i^2}{4 B_i} + \frac{\mathbf{x}_i (3 \gamma \mathcal{A}_i^2 - \gamma B_i \mathcal{A}_i^2) \beta_i \eta_i \xi_i^2}{B_i} + \frac{\mathbf{a}_i (-3 \gamma \mathcal{A}_i^2 + 2 \gamma B_i \mathcal{A}_i^2) \eta_i^2 \xi_i^2}{2 B_i^2} + \\
 & \frac{(17 \gamma^2 \mathcal{A}_i^2 - 22 \gamma^2 B_i \mathcal{A}_i^2 + 5 \gamma^2 B_i^2 \mathcal{A}_i^2) \eta_i^2 \xi_i^2}{8 B_i^2} + \frac{\mathbf{x}_i y_i (21 \gamma^2 h \mathcal{A}_i^2 - 5 \gamma^2 h B_i \mathcal{A}_i^2) \eta_i^2 \xi_i^2}{4 B_i^2} + \frac{(-3 \gamma \mathcal{A}_i^2 + 4 \gamma B_i \mathcal{A}_i^2 - \gamma B_i^2 \mathcal{A}_i^2) \beta_i \eta_i^2 \xi_i^2}{2 h B_i^2} + \\
 & \frac{y_i (-41 \gamma^2 \mathcal{A}_i^2 + 34 \gamma^2 B_i \mathcal{A}_i^2 - 5 \gamma^2 B_i^2 \mathcal{A}_i^2) \eta_i^3 \xi_i^2}{12 B_i^2} - \frac{1}{2} \gamma^2 h^2 \mathbf{x}_i^3 \mathcal{A}_i^3 \xi_i^3 + \frac{\mathbf{x}_i^2 (8 \gamma^2 h \mathcal{A}_i^2 - 2 \gamma^2 h B_i \mathcal{A}_i^2) \eta_i \xi_i^2}{3 B_i} + \\
 & \left. \frac{\mathbf{x}_i (-41 \gamma^2 \mathcal{A}_i^2 + 34 \gamma^2 B_i \mathcal{A}_i^2 - 5 \gamma^2 B_i^2 \mathcal{A}_i^2) \eta_i^2 \xi_i^2}{12 B_i^2} + \frac{(41 \gamma^2 \mathcal{A}_i^2 - 75 \gamma^2 B_i \mathcal{A}_i^2 + 39 \gamma^2 B_i^2 \mathcal{A}_i^2 - 5 \gamma^2 B_i^3 \mathcal{A}_i^2) \eta_i^3 \xi_i^2}{36 h B_i^2} \right) \in^2 + \\
 & \left( \frac{\gamma^3 h^3 y_i \mathcal{A}_i \eta_i}{6 B_i} - \frac{\gamma^2 h^2 y_i \mathcal{A}_i \beta_i \eta_i}{2 B_i} + \frac{\gamma h y_i \mathcal{A}_i \beta_i^2 \eta_i}{2 B_i} - \frac{y_i \mathcal{A}_i \beta_i^3 \eta_i}{6 B_i} - \frac{19 \gamma^2 h^3 y_i^2 \mathcal{A}_i^2 \eta_i^2}{12 B_i^2} + \frac{5 \gamma^2 h^2 y_i^3 \mathcal{A}_i^3 \eta_i^3}{2 B_i^2} - \frac{\gamma h y_i^2 \mathcal{A}_i^2 \beta_i \eta_i^2}{B_i^2} + \right. \\
 & \frac{13 \gamma^3 h^3 y_i^3 \mathcal{A}_i^3 \eta_i^3}{6 B_i^2} - \frac{3 \gamma^2 h^2 y_i^2 \mathcal{A}_i^2 \beta_i \eta_i^2}{2 B_i^2} - \frac{2 \gamma^3 h^3 y_i^4 \mathcal{A}_i^4 \eta_i^4}{3 B_i^2} - \frac{1}{6} h^3 \mathbf{a}_i^3 \mathbf{x}_i \mathcal{A}_i \xi_i - \frac{1}{2} h^2 \mathbf{a}_i^2 \mathbf{x}_i \mathcal{A}_i \beta_i \xi_i - \\
 & \frac{1}{2} h \mathbf{a}_i \mathbf{x}_i \mathcal{A}_i \beta_i^2 \xi_i - \frac{1}{6} \mathbf{x}_i \mathcal{A}_i \beta_i^3 \xi_i + \frac{\gamma^2 h^2 \mathbf{a}_i \mathcal{A}_i \eta_i \xi_i}{2 B_i} - \frac{\gamma h^2 \mathbf{a}_i^2 \mathcal{A}_i \eta_i \xi_i}{2 B_i} + \frac{h^2 \mathbf{a}_i^3 \mathcal{A}_i \eta_i \xi_i}{6 B_i} - \frac{7 \gamma^3 h^3 \mathbf{x}_i y_i \mathcal{A}_i^2 \eta_i \xi_i}{6 B_i} + \\
 & \frac{3 \gamma^2 h^3 \mathbf{a}_i \mathbf{x}_i y_i \mathcal{A}_i^2 \eta_i \xi_i}{2 B_i} - \frac{\gamma h^3 \mathbf{a}_i^2 \mathbf{x}_i y_i \mathcal{A}_i^2 \eta_i \xi_i}{2 B_i} + \frac{(-\gamma^3 h^2 \mathcal{A}_i + \gamma^3 h^2 B_i \mathcal{A}_i) \eta_i \xi_i}{6 B_i} - \frac{\gamma h \mathbf{a}_i \mathcal{A}_i \beta_i \eta_i \xi_i}{B_i} + \frac{h \mathbf{a}_i^2 \mathcal{A}_i \beta_i \eta_i \xi_i}{2 B_i} + \\
 & \frac{3 \gamma^2 h^2 \mathbf{x}_i y_i \mathcal{A}_i^2 \beta_i \eta_i \xi_i}{B_i} - \frac{2 \gamma h^2 \mathbf{a}_i \mathbf{x}_i y_i \mathcal{A}_i^2 \beta_i \eta_i \xi_i}{B_i} + \frac{(\gamma^2 h \mathcal{A}_i - \gamma^2 h B_i \mathcal{A}_i) \beta_i \eta_i \xi_i}{2 B_i} + \frac{\mathbf{a}_i \mathcal{A}_i \beta_i^2 \eta_i \xi_i}{2 B_i} - \frac{2 \gamma h \mathbf{x}_i y_i \mathcal{A}_i^2 \beta_i^2 \eta_i \xi_i}{B_i} + \\
 & \frac{(-\gamma \mathcal{A}_i + \gamma B_i \mathcal{A}_i) \beta_i^2 \eta_i \xi_i}{2 B_i} + \frac{(\mathcal{A}_i - B_i \mathcal{A}_i) \beta_i^2 \eta_i \xi_i}{6 h B_i} - \frac{17 \gamma^2 h^2 \mathbf{a}_i y_i \mathcal{A}_i^2 \eta_i^2 \xi_i}{4 B_i^2} + \frac{3 \gamma h^2 \mathbf{a}_i^2 y_i \mathcal{A}_i^2 \eta_i^2 \xi_i}{4 B_i^2} + \frac{5 \gamma^3 h^3 \mathbf{x}_i y_i^2 \mathcal{A}_i^2 \eta_i^2 \xi_i}{B_i^2} - \\
 & \frac{3 \gamma^2 h^3 \mathbf{a}_i \mathbf{x}_i y_i^2 \mathcal{A}_i^2 \eta_i^2 \xi_i}{2 B_i^2} + \frac{y_i (75 \gamma^3 h^2 \mathcal{A}_i^2 - 19 \gamma^3 h^2 B_i \mathcal{A}_i^2) \eta_i^2 \xi_i}{12 B_i^2} + \frac{3 \gamma h \mathbf{a}_i y_i \mathcal{A}_i^2 \beta_i \eta_i^2 \xi_i}{B_i^2} - \frac{9 \gamma^2 h^2 \mathbf{x}_i y_i^2 \mathcal{A}_i^2 \beta_i \eta_i^2 \xi_i}{2 B_i^2} + \\
 & \frac{y_i (-17 \gamma^2 h \mathcal{A}_i^2 + 5 \gamma^2 h B_i \mathcal{A}_i^2) \beta_i \eta_i^2 \xi_i}{2 B_i^2} + \frac{y_i (3 \gamma \mathcal{A}_i^2 - \gamma B_i \mathcal{A}_i^2) \beta_i^2 \eta_i^2 \xi_i}{B_i^2} + \frac{8 \gamma^2 h^2 \mathbf{a}_i y_i^2 \mathcal{A}_i^2 \eta_i^2 \xi_i}{3 B_i^2} - \frac{8 \gamma^3 h^3 \mathbf{x}_i y_i^2 \mathcal{A}_i^2 \eta_i^2 \xi_i}{3 B_i^2} + \\
 & \frac{y_i^2 (-79 \gamma^3 h^2 \mathcal{A}_i^2 + 17 \gamma^3 h^2 B_i \mathcal{A}_i^2) \eta_i^3 \xi_i}{6 B_i^2} + \frac{y_i^2 (8 \gamma^2 h \mathcal{A}_i^2 - 2 \gamma^2 h B_i \mathcal{A}_i^2) \beta_i \eta_i^3 \xi_i}{B_i^2} + \frac{y_i^2 (125 \gamma^3 h^2 \mathcal{A}_i^2 - 27 \gamma^3 h^2 B_i \mathcal{A}_i^2) \eta_i^4 \xi_i}{24 B_i^2} - \\
 & \frac{1}{12} \gamma^3 h^3 \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^2 + \frac{1}{2} \gamma^2 h^3 \mathbf{a}_i \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^2 - \gamma h^3 \mathbf{a}_i^2 \mathbf{x}_i^2 \mathcal{A}_i^2 \xi_i^2 + \frac{1}{2} \gamma^2 h^2 \mathbf{x}_i^2 \mathcal{A}_i^2 \beta_i \xi_i^2 - 2 \gamma h^2 \mathbf{a}_i \mathbf{x}_i^2 \mathcal{A}_i^2 \beta_i \xi_i^2 - \\
 & \gamma h \mathbf{x}_i^2 \mathcal{A}_i^2 \beta_i^2 \xi_i^2 + \frac{7 \gamma^3 h^3 \mathbf{x}_i^2 y_i \mathcal{A}_i^2 \eta_i \xi_i^2}{2 B_i} - \frac{3 \gamma^2 h^3 \mathbf{a}_i \mathbf{x}_i^2 y_i \mathcal{A}_i^2 \eta_i \xi_i^2}{B_i} + \frac{\mathbf{a}_i^2 \mathbf{x}_i (12 \gamma h^2 \mathcal{A}_i^2 - \gamma h^2 B_i \mathcal{A}_i^2) \eta_i \xi_i^2}{4 B_i} + \\
 & \frac{\mathbf{a}_i \mathbf{x}_i (-22 \gamma^2 h^2 \mathcal{A}_i^2 + 3 \gamma^2 h^2 B_i \mathcal{A}_i^2) \eta_i \xi_i^2}{4 B_i} + \frac{\mathbf{x}_i (33 \gamma^3 h^2 \mathcal{A}_i^2 - 7 \gamma^3 h^2 B_i \mathcal{A}_i^2) \eta_i \xi_i^2}{12 B_i} - \frac{9 \gamma^2 h^2 \mathbf{x}_i^2 y_i \mathcal{A}_i^2 \beta_i \eta_i \xi_i^2}{2 B_i} + \\
 & \frac{\mathbf{a}_i \mathbf{x}_i (6 \gamma h \mathcal{A}_i^2 - \gamma h B_i \mathcal{A}_i^2) \beta_i \eta_i \xi_i^2}{B_i} + \frac{\mathbf{x}_i (-11 \gamma^2 h \mathcal{A}_i^2 + 3 \gamma^2 h B_i \mathcal{A}_i^2) \beta_i \eta_i \xi_i^2}{2 B_i} + \frac{\mathbf{x}_i (3 \gamma \mathcal{A}_i^2 - \gamma B_i \mathcal{A}_i^2) \beta_i^2 \eta_i \xi_i^2}{B_i} - \frac{4 \gamma^3 h^3 \mathbf{x}_i^2 y_i^2 \mathcal{A}_i^2 \eta_i^2 \xi_i^2}{B_i^2} + \\
 & \frac{\mathbf{a}_i^2 (-3 \gamma h \mathcal{A}_i^2 + \gamma h B_i \mathcal{A}_i^2) \eta_i^2 \xi_i^2}{2 B_i^2} + \frac{\mathbf{a}_i (17 \gamma^2 h \mathcal{A}_i^2 - 11 \gamma^2 h B_i \mathcal{A}_i^2) \eta_i^2 \xi_i^2}{4 B_i^2} + \frac{(-75 \gamma^3 h \mathcal{A}_i^2 + 94 \gamma^3 h B_i \mathcal{A}_i^2 - 19 \gamma^3 h B_i^2 \mathcal{A}_i^2) \eta_i^2 \xi_i^2}{24 B_i^2} + \\
 & \frac{\mathbf{a}_i \mathbf{x}_i y_i (42 \gamma^2 h^2 \mathcal{A}_i^2 - 5 \gamma^2 h^2 B_i \mathcal{A}_i^2) \eta_i^2 \xi_i^2}{4 B_i^2} + \frac{\mathbf{x}_i y_i (-41 \gamma^3 h^2 \mathcal{A}_i^2 + 8 \gamma^3 h^2 B_i \mathcal{A}_i^2) \eta_i^2 \xi_i^2}{2 B_i^2} + \frac{\mathbf{a}_i (-3 \gamma \mathcal{A}_i^2 + 2 \gamma B_i \mathcal{A}_i^2) \beta_i \eta_i^2 \xi_i^2}{B_i^2} + \\
 & \frac{(17 \gamma^2 \mathcal{A}_i^2 - 22 \gamma^2 B_i \mathcal{A}_i^2 + 5 \gamma^2 B_i^2 \mathcal{A}_i^2) \beta_i \eta_i^2 \xi_i^2}{4 B_i^2} + \frac{\mathbf{x}_i y_i (63 \gamma^2 h \mathcal{A}_i^2 - 15 \gamma^2 h B_i \mathcal{A}_i^2) \beta_i \eta_i^2 \xi_i^2}{4 B_i^2} + \frac{(-3 \gamma \mathcal{A}_i^2 + 4 \gamma B_i \mathcal{A}_i^2 - \gamma B_i^2 \mathcal{A}_i^2) \beta_i^2 \eta_i^2 \xi_i^2}{2 h B_i^2} + \\
 & \frac{\mathbf{a}_i y_i (-41 \gamma^2 h \mathcal{A}_i^2 + 17 \gamma^2 h B_i \mathcal{A}_i^2) \eta_i^3 \xi_i^2}{6 B_i^2} + \frac{y_i (71 \gamma^3 h \mathcal{A}_i^2 - 54 \gamma^3 h B_i \mathcal{A}_i^2 + 7 \gamma^3 h B_i^2 \mathcal{A}_i^2) \eta_i^3 \xi_i^2}{4 B_i^2} + \frac{\mathbf{x}_i y_i^2 (92 \gamma^3 h^2 \mathcal{A}_i^2 - 19 \gamma^3 h^2 B_i \mathcal{A}_i^2) \eta_i^3 \xi_i^2}{6 B_i^2} + \\
 & \frac{y_i (-41 \gamma^2 \mathcal{A}_i^2 + 34 \gamma^2 B_i \mathcal{A}_i^2 - 5 \gamma^2 B_i^2 \mathcal{A}_i^2) \beta_i \eta_i^3 \xi_i^2}{4 B_i^2} + \frac{y_i^2 (-557 \gamma^3 h \mathcal{A}_i^2 + 360 \gamma^3 h B_i \mathcal{A}_i^2 - 43 \gamma^3 h B_i^2 \mathcal{A}_i^2) \eta_i^4 \xi_i^2}{48 B_i^2} + \\
 & \frac{2}{3} \gamma^3 h^3 \mathbf{x}_i^3 \mathcal{A}_i^3 \xi_i^3 - \frac{3}{2} \gamma^2 h^3 \mathbf{a}_i \mathbf{x}_i^3 \mathcal{A}_i^3 \xi_i^3 - \frac{3}{2} \gamma^2 h^2 \mathbf{x}_i^3 \mathcal{A}_i^3 \beta_i \xi_i^3 - \frac{8 \gamma^3 h^3 \mathbf{x}_i^2 y_i \mathcal{A}_i^2 \eta_i \xi_i^2}{3 B_i} + \\
 & \frac{\mathbf{a}_i \mathbf{x}_i^2 (24 \gamma^2 h^2 \mathcal{A}_i^2 - 12 \gamma^2 h^2 B_i \mathcal{A}_i^2) \eta_i \xi_i^2}{3 B_i} + \frac{\mathbf{x}_i^2 (43 \gamma^3 h^2 \mathcal{A}_i^2 - 11 \gamma^3 h^2 B_i \mathcal{A}_i^2) \eta_i \xi_i^2}{6 B_i} + \frac{\mathbf{a}_i \mathbf{x}_i^2 (12 \gamma h \mathcal{A}_i^2 - \gamma h B_i \mathcal{A}_i^2) \beta_i \eta_i \xi_i^2}{B_i} + \\
 & \left. \frac{\mathbf{a}_i \mathbf{x}_i (-123 \gamma^2 h \mathcal{A}_i^2 + 68 \gamma^2 h B_i \mathcal{A}_i^2 - 5 \gamma^2 h B_i^2 \mathcal{A}_i^2) \eta_i^2 \xi_i^2}{12 B_i^2} + \frac{\mathbf{x}_i (43 \gamma^3 h \mathcal{A}_i^2 - 32 \gamma^3 h B_i \mathcal{A}_i^2 + 4 \gamma^3 h B_i^2 \mathcal{A}_i^2) \eta_i^2 \xi_i^2}{3 B_i^2} \right)
 \end{aligned}$$

$$\begin{aligned} a\Delta \rightarrow \mathbb{E}\mathbb{E}_{(i) \rightarrow \{j,k\}} & \left[ a_j \alpha_i + a_k \alpha_i, x_j \xi_i + x_k \xi_i, \left( -\hbar a_j x_k \xi_i + \frac{1}{2} \gamma \hbar x_j x_k \xi_i^2 \right) \epsilon + \right. \\ & \left( \frac{1}{2} \hbar^2 a_j^2 x_k \xi_i + \frac{1}{4} \gamma^2 \hbar^2 x_j x_k \xi_i^2 - \frac{1}{2} \gamma \hbar^2 a_j x_j x_k \xi_i^2 + \frac{1}{6} \gamma^2 \hbar^2 x_j^2 x_k \xi_i^3 + \frac{1}{6} \gamma^2 \hbar^2 x_j x_k^2 \xi_i^3 \right) \epsilon^2 + \\ & \left( -\frac{1}{6} \hbar^3 a_j^3 x_k \xi_i + \frac{1}{12} \gamma^3 \hbar^3 x_j x_k \xi_i^2 - \frac{1}{4} \gamma^2 \hbar^3 a_j x_j x_k \xi_i^2 + \frac{1}{4} \gamma \hbar^3 a_j^2 x_j x_k \xi_i^2 + \right. \\ & \quad \frac{1}{6} \gamma^3 \hbar^3 x_j^2 x_k \xi_i^3 - \frac{1}{6} \gamma^2 \hbar^3 a_j x_j^2 x_k \xi_i^3 + \frac{1}{6} \gamma^3 \hbar^3 x_j x_k^2 \xi_i^3 - \frac{1}{3} \gamma^2 \hbar^3 a_j x_j x_k^2 \xi_i^3 + \\ & \quad \left. \frac{1}{24} \gamma^3 \hbar^3 x_j^3 x_k \xi_i^4 + \frac{5}{24} \gamma^3 \hbar^3 x_j^2 x_k^2 \xi_i^4 + \frac{1}{24} \gamma^3 \hbar^3 x_j x_k^3 \xi_i^4 \right) \epsilon^3 + \mathcal{O}[\epsilon^4] \end{aligned}$$

$$\begin{aligned} b\Delta \rightarrow \mathbb{E}\mathbb{E}_{(i) \rightarrow \{j,k\}} & \left[ b_j \beta_i + b_k \beta_i, B_k y_j \eta_i + y_k \eta_i, \right. \\ & \frac{1}{2} \gamma \hbar B_k y_j y_k \eta_i^2 \epsilon + \left( \frac{1}{4} \gamma^2 \hbar^2 B_k y_j y_k \eta_i^2 + \frac{1}{6} \gamma^2 \hbar^2 B_k^2 y_j^2 y_k \eta_i^2 + \frac{1}{6} \gamma^2 \hbar^2 B_k y_j y_k^2 \eta_i^2 \right) \epsilon^2 + \\ & \left( \frac{1}{12} \gamma^3 \hbar^3 B_k y_j y_k \eta_i^2 + \frac{1}{6} \gamma^3 \hbar^3 B_k^2 y_j^2 y_k \eta_i^2 + \frac{1}{6} \gamma^3 \hbar^3 B_k y_j y_k^2 \eta_i^2 + \right. \\ & \quad \left. \frac{1}{24} \gamma^3 \hbar^3 B_k^3 y_j^3 y_k \eta_i^4 + \frac{5}{24} \gamma^3 \hbar^3 B_k^2 y_j^2 y_k^2 \eta_i^4 + \frac{1}{24} \gamma^3 \hbar^3 B_k y_j y_k^3 \eta_i^4 \right) \epsilon^3 + \mathcal{O}[\epsilon^4] \end{aligned}$$

$$\begin{aligned} d\Delta \rightarrow \mathbb{E}\mathbb{E}_{(i) \rightarrow \{j,k\}} & \left[ a_j \alpha_i + a_k \alpha_i + b_j \beta_i + b_k \beta_i, \right. \\ & y_j \eta_i + B_j y_k \eta_i + x_j \xi_i + x_k \xi_i, \left( \frac{1}{2} \gamma \hbar B_j y_j y_k \eta_i^2 - \hbar a_j x_k \xi_i + \frac{1}{2} \gamma \hbar x_j x_k \xi_i^2 \right) \epsilon + \\ & \left( \frac{1}{4} \gamma^2 \hbar^2 B_j y_j y_k \eta_i^2 + \frac{1}{6} \gamma^2 \hbar^2 B_j y_j^2 y_k \eta_i^2 + \frac{1}{6} \gamma^2 \hbar^2 B_j^2 y_j y_k^2 \eta_i^2 + \frac{1}{2} \hbar^2 a_j^2 x_k \xi_i + \right. \\ & \quad \left. \frac{1}{4} \gamma^2 \hbar^2 x_j x_k \xi_i^2 - \frac{1}{2} \gamma \hbar^2 a_j x_j x_k \xi_i^2 + \frac{1}{6} \gamma^2 \hbar^2 x_j^2 x_k \xi_i^3 + \frac{1}{6} \gamma^2 \hbar^2 x_j x_k^2 \xi_i^3 \right) \epsilon^2 + \\ & \left( \frac{1}{12} \gamma^3 \hbar^3 B_j y_j y_k \eta_i^2 + \frac{1}{6} \gamma^3 \hbar^3 B_j y_j^2 y_k \eta_i^2 + \frac{1}{6} \gamma^3 \hbar^3 B_j^2 y_j y_k^2 \eta_i^2 + \frac{1}{24} \gamma^3 \hbar^3 B_j y_j^3 y_k \eta_i^4 + \right. \\ & \quad \frac{5}{24} \gamma^3 \hbar^3 B_j^2 y_j^2 y_k^2 \eta_i^4 + \frac{1}{24} \gamma^3 \hbar^3 B_j^3 y_j y_k^3 \eta_i^4 - \frac{1}{6} \hbar^3 a_j^3 x_k \xi_i + \frac{1}{12} \gamma^3 \hbar^3 x_j x_k \xi_i^2 - \\ & \quad \frac{1}{4} \gamma^2 \hbar^3 a_j x_j x_k \xi_i^2 + \frac{1}{4} \gamma \hbar^3 a_j^2 x_j x_k \xi_i^2 + \frac{1}{6} \gamma^3 \hbar^3 x_j^2 x_k \xi_i^3 - \frac{1}{6} \gamma^2 \hbar^3 a_j x_j^2 x_k \xi_i^3 + \frac{1}{6} \gamma^3 \hbar^3 x_j x_k^2 \xi_i^3 - \\ & \quad \left. \frac{1}{3} \gamma^2 \hbar^3 a_j x_j x_k^2 \xi_i^3 + \frac{1}{24} \gamma^3 \hbar^3 x_j^3 x_k \xi_i^4 + \frac{5}{24} \gamma^3 \hbar^3 x_j^2 x_k^2 \xi_i^4 + \frac{1}{24} \gamma^3 \hbar^3 x_j x_k^3 \xi_i^4 \right) \epsilon^3 + \mathcal{O}[\epsilon^4] \end{aligned}$$

$$C \rightarrow \mathbb{E}\mathbb{E}_{\{\} \rightarrow \{i\}} \left[ \emptyset, \emptyset, \frac{\text{Log}[B_i]}{2} - \frac{1}{2} (\hbar a_i) \epsilon + \mathcal{O}[\epsilon^4] \right]$$

$$\bar{C} \rightarrow \mathbb{E}\mathbb{E}_{\{\} \rightarrow \{i\}} \left[ \emptyset, \emptyset, -\frac{\text{Log}[B_i]}{2} + \frac{1}{2} \hbar a_i \epsilon + \mathcal{O}[\epsilon^4] \right]$$

$$\begin{aligned} \text{Kink} \rightarrow \mathbb{E}\mathbb{E}_{\{\} \rightarrow \{i\}} & \left[ \hbar a_i b_i, \hbar x_i y_i, \right. \\ & \left. -\frac{\text{Log}[B_i]}{2} + \left( \frac{\hbar a_i}{2} - \frac{1}{4} \gamma \hbar^3 x_i^2 y_i^2 \right) \epsilon + \frac{1}{9} \gamma^2 \hbar^5 x_i^3 y_i^3 \epsilon^2 + \left( \frac{1}{48} \gamma^3 \hbar^5 x_i^2 y_i^2 - \frac{1}{16} \gamma^3 \hbar^7 x_i^4 y_i^4 \right) \epsilon^3 + \mathcal{O}[\epsilon^4] \right] \end{aligned}$$

$$\begin{aligned} \overline{\text{Kink}} \rightarrow \mathbb{E}\mathbb{E}_{\{\} \rightarrow \{i\}} & \left[ -\hbar a_i b_i, -\frac{\hbar x_i y_i}{B_i}, \right. \\ & \frac{\text{Log}[B_i]}{2} + \left( -\frac{\hbar a_i}{2} - \frac{\hbar^2 a_i x_i y_i}{B_i} - \frac{3 \gamma \hbar^3 x_i^2 y_i^2}{4 B_i^2} \right) \epsilon + \left( -\frac{\hbar^3 a_i^2 x_i y_i}{2 B_i} + \frac{\gamma^2 \hbar^4 x_i^2 y_i^2}{2 B_i^2} - \frac{3 \gamma \hbar^4 a_i x_i y_i^2}{2 B_i^2} - \frac{10 \gamma^2 \hbar^5 x_i^3 y_i^3}{9 B_i^3} \right) \epsilon^2 + \\ & \left( -\frac{\hbar^4 a_i^3 x_i y_i}{6 B_i} - \frac{3 \gamma^3 \hbar^5 x_i^2 y_i^2}{16 B_i^2} + \frac{\gamma^2 \hbar^5 a_i x_i^2 y_i^2}{B_i^2} - \frac{3 \gamma \hbar^5 a_i^2 x_i^2 y_i^2}{2 B_i^2} + \frac{2 \gamma^3 \hbar^6 x_i^3 y_i^3}{B_i^3} - \frac{10 \gamma^2 \hbar^6 a_i x_i^3 y_i^3}{3 B_i^3} - \frac{35 \gamma^3 \hbar^7 x_i^4 y_i^4}{16 B_i^4} \right) \epsilon^3 + \mathcal{O}[\epsilon^4] \end{aligned}$$

$$b2t \rightarrow \mathbb{E}\mathbb{E}_{(i) \rightarrow \{i\}} \left[ a_i \alpha_i - \frac{t_i \beta_i}{\gamma}, y_i \eta_i + x_i \xi_i, \frac{a_i \beta_i \epsilon}{\gamma} + \mathcal{O}[\epsilon^4] \right]$$

$$t2b \rightarrow \mathbb{E}\mathbb{E}_{(i) \rightarrow \{i\}} \left[ a_i \alpha_i - \gamma b_i \tau_i, y_i \eta_i + x_i \xi_i, a_i \tau_i \epsilon + \mathcal{O}[\epsilon^4] \right]$$

In[\*]= **degs = atoms / .**  $\mathbb{E}_{sp\_} [L\_ , Q\_ , P\_ ] \Rightarrow$

**Exponent** [**Normal**@**CF**@**Log**@**P** / . { (v : x | y |  $\xi$  |  $\eta$ )<sub>i</sub>  $\Rightarrow$   $\lambda v_i$ , (v : a |  $\beta$ )<sub>i</sub>  $\Rightarrow$   $\lambda^2 v_i$ },  $\lambda$ ]

Out[\*]= { am  $\rightarrow$   $-\infty$ , bm  $\rightarrow$  6, dm  $\rightarrow$  6, R  $\rightarrow$  4,  $\bar{R}$   $\rightarrow$  4, P  $\rightarrow$  4, aS  $\rightarrow$  8,  $\bar{aS}$   $\rightarrow$  8, bS  $\rightarrow$  8,  $\bar{bS}$   $\rightarrow$  8, dS  $\rightarrow$  8, a $\Delta$   $\rightarrow$  4, b $\Delta$   $\rightarrow$  4, d $\Delta$   $\rightarrow$  4, C  $\rightarrow$  2,  $\bar{C}$   $\rightarrow$  2, Kink  $\rightarrow$  8,  $\overline{\text{Kink}}$   $\rightarrow$  8, b2t  $\rightarrow$  4, t2b  $\rightarrow$  2 }

In[\*]= **Last /@ degs**

Out[\*]= {  $-\infty$ , 6, 6, 4, 4, 4, 8, 8, 8, 8, 8, 4, 4, 4, 2, 2, 8, 8, 4, 2 }

In[\*]= **Max [Last /@ degs]**

Out[\*]= 8